Getting Away with Robbery? Patenting Behavior with the Threat of Infringement

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Abstract
The paper examines how the innovator's ability to enforce her patent rights affects (and is affected by) her decision to patent her innovation and her patent breadth decision. Specifically, the paper shows that the innovator might find it optimal to patent her innovation even if the patent would not be defended if it were violated. Nevertheless, the patent is valuable because it can be used to influence the entrant's location decision in a way that is profitable for the incumbent. In addition to showing that a patent need not be actually enforced when infringed to be valuable, the paper shows that the greater is the entrant's R&D effectiveness, the smaller is the innovator's incentive to patent her product. If patenting occurs, however, the greater is R&D effectiveness, the greater is the patent breadth that could be chosen without triggering infringement.

Keywords: patent breadth, entry deterrence, patent infringement, patent validity.

JEL Classification Codes: L20; L13; O34.

1. Introduction

The decision to patent an innovation suggests that the innovator could then strategically use the resulting intellectual property right to affect market entry and the location decisions of potential entrants, thereby increasing profits.[1] For example, Gallini (1984) shows that an incumbent monopoly may license a technology to an entrant, since allowing her into the market reduces her incentive to undertake R&D that could make her an even stronger competitor. In a similar vein, Rockett (1990) argues that licensing can be used as a way of selecting weak competitors; their presence then keeps stronger competitors out of the market when the patent expires. Yiannaka and

[1] For an analysis of the innovator's decision to patent her innovation or to keep it a secret, see Horstmann et al. (1985), Waterson (1990), Aoki and Spiegel (2003), and Erkal (2005).

Fulton (2006) show that an incumbent can use her patent breadth to alter the product location of an entrant in a way that increases her profits.

In the above cases, the strategic value of the patent is its ability to influence the entrant's entry and location behavior, since the patent, which is costly to obtain, will have no value if no change in behavior is induced. The ability to strategically use the patent to influence behavior depends critically on its enforceability; if it cannot be enforced when infringed, then no change in behavior can be achieved.[2] However, as we posit in this paper, a patent need not be actually enforced when infringed to be valuable; instead, a patent can be valuable if it is potentially enforceable.

The purpose of our paper is twofold. First, the paper examines how the innovator's ability to enforce her patent rights affects and is affected by her decision to patent her innovation and her patent breadth decision. Second, the paper shows that a patent that is not actually enforced can still be valuable for the patentee by inducing the desired behavior by the entrant. To address these issues, the paper develops a game theoretic model that examines the optimal patenting behavior of an incumbent innovator who has generated a patentable product innovation and who is faced with potential entry by an entrant supplying a superior quality product. The incumbent/innovator has to decide whether she should patent her innovation and, if so, what patent breadth should be claimed. If her patent is infringed, the incumbent also has to decide whether she should invoke a trial to defend the patent. An important feature of the model is that the entrant may be able, by his choice of location in product space, to affect the incumbent's decision to defend her patent.

Using a general model, the paper gives the conditions under which the patent has no impact on the entrant's behavior and thus patenting is not desirable. These conditions are associated with relatively small R&D costs for the potential entrant and large trial costs and small monopoly profits

[2] Crampes and Langinier (2002) examine the patentee's optimal reaction in the case of infringement – to go to court, to settle or to accept entry – without considering, however, the decision to patent or the patent breadth decision.

for the innovator. When the above conditions do not obtain, a patent, combined with the optimal patent breadth, can affect the entrant's entry and location decisions and patenting may be optimal for the innovator. When patenting is optimal, the smaller are the entrant's R&D costs, the greater is the patent breadth that could be chosen without triggering infringement. This result occurs because the greater is the entrant's R&D effectiveness, the further away from the incumbent the entrant can locate in the product space; the outcome is increased product differentiation, less competition and thus higher profits for both players. Moreover, there are certain conditions when the innovator can use the breadth of her patent to deter market entry – specifically when the entrant's R&D and trial costs are large, the innovator's trial costs are small and her monopoly profits are large and the effect of patent breadth on patent validity is small.

A key finding of the paper is that the innovator might find it optimal to patent her innovation even if the patent would not be defended if it were to be violated. This result (which is more likely to occur when the entrant's R&D effectiveness is relatively high) occurs because, by choosing to patent her innovation, the incumbent can induce the entrant to choose a location in product space that, even though it infringes the patent and is not enforced, is still more advantageous for the incumbent than the other possible outcomes. Under this case, the entrant, knowing that his location decision affects the incumbent's decision to invoke a trial, strategically chooses a location that will not be challenged by the incumbent. The possibility that an incumbent might patent an innovation, even though she would not legally enforce it, depends critically on the fact that the patent is potentially enforceable; it is this potential enforceability that allows the incumbent to affect the entrant's location decision who is induced to choose a location that will not be challenged. Without this enforceability, the entrant would simply locate at his most preferred location (where he would have located under no patent protection) and not the one desired by the incumbent.

The rest of the paper is organized as follows. Section 2 describes the theoretical development of the patenting decisions model (i.e., the decision to patent, the patent breadth decision and the decision to invoke a trial under infringement), section 3 provides the analytical solution of the patenting game and section 4 concludes the paper. An application of the general model is provided in the Appendix.

2. The patenting decisions model

2.1 Model assumptions

Our model builds upon the model developed by Yiannaka and Fulton (2006) who study the optimal patent breadth decision when under infringement a trial always takes place. In addition to examining the innovator's optimal patent breadth decision, our model considers the innovator's decision to patent and her decision to invoke a trial when her patent is infringed. The patenting decisions are modeled in a sequential game of complete and perfect information between two agents; an incumbent innovator who has invented a patentable drastic product innovation and a potential entrant. At the beginning of the game the incumbent's product has already been generated.[3] The incumbent decides whether to seek patent protection, how broad of a protection to claim and whether to defend her patent when infringement occurs; the entrant decides whether to enter the incumbent's market and, if entry occurs, where to locate in a vertically differentiated product space. To keep the focus on the innovator's patenting and patent breadth decisions we assume that the regulator (e.g., Patent Office) always grants the patent as claimed; thus, the regulator is not explicitly modeled.[4]

[3] Even thought the innovator's R&D investment decision that led to the generation of the innovation is important, to keep the analysis tractable, this decision is not considered here.

[4] As in Yiannaka and Fulton (2006), who point to failures in the patent granting process and suggest that the innovator cannot always rely on the Patent Office for help in refining her patent claims, we examine the innovator's patenting behavior when she has no feedback from the Patent Office.

The incumbent and the entrant, if he enters, operate in a vertically differentiated product market and produce qualities $q_p = 0$ and q_e, respectively, where the entrant's product is the superior quality product, $q_e > q_p = 0$.[5] The maximum distance the entrant can locate away from the incumbent is normalized to equal one, $q_e \in (0,1]$. It is further assumed that each of the agents produces at most one product for which no other substitute exists, consumers buy one unit of either the incumbent's or the entrant's product and the entrant does not patent his product since further entry is not considered. Both agents are risk neutral and maximize profits.

The incumbent's decision to patent the innovation implies patenting costs denoted by z ($z > 0$), that are assumed to be independent of patent breadth. This assumption is in line with our assumption that the Patent Office always grants the patent as claimed. At the beginning of the game the incumbent's R&D costs, denoted by F_p, are sunk. The entrant's R&D costs of developing the higher quality product are given by $F_e(q_e)$, where $F_e'(q_e) > 0$, $F_e''(q_e) > 0$ and $F_e(q_p) = 0$. The above imply that it is increasingly costly for the entrant to locate away from the incumbent in the one-dimensional product space (i.e., to produce the better quality product) and the filing of a patent by the incumbent provides the entrant with knowledge of how to produce the incumbent's product (i.e., the assumption of perfect information disclosure by the patent is made). An important assumption of the model is that, in the absence of patent protection, reverse engineering of the product innovation is possible and costless. Once the R&D costs are incurred, production of the products by both the incumbent and the entrant occur at zero marginal cost and neither the incumbent nor the entrant find it optimal to relocate once they have chosen their respective qualities (i.e., relocation is prohibitively costly).

[5] Setting the quality of the incumbent's product q_p equal to zero simplifies the notation without affecting the qualitative nature of the model. As a result, the entrant's quality q_e is interpreted as the difference in quality between his product and that of the incumbent, or more generally as the distance the entrant has located away from the incumbent.

The patent breadth is denoted by b where $b \in (0,1]$ determines the area in the one-dimensional product space that the patent protects. Patent breadth thus determines the minimum size of q_e needed to ensure that the entrant's product does not infringe on the incumbent's patent. A fencepost patent system is assumed where patent claims define an exact border of protection and where infringement will always be found when an entrant locates within the incumbent's claims, unless the entrant proves that the patent is invalid (Cornish 1989).[6]

The probability that the patent will be found to be valid, or equivalently that infringement will be found, is given by $\mu(b,\alpha)$ where $\alpha \in (0,1)$, $\mu(b \to 0) \to 1$, $\mu(b=1) \neq 0$, $\mu_b < 0$, $\mu_{bb} \leq 0$, $\mu_\alpha < 0$, $\mu_{\alpha\alpha} \leq 0$ and $\mu_{b\alpha} < 0$. The parameter α is a validity parameter that reflects the degree that patent breadth affects patent validity; for any given patent breadth, the greater is the validity parameter, the smaller is the probability that the patent will be found valid. The inverse relationship between the probability that the patent will be found valid and patent breadth is based on evidence from the literature that shows that, the broader is the patent protection, the harder it is to establish validity since the harder it is to show novelty, nonobviousness and enablement (Cornish 1989, Miller and Davis 1990). In addition, courts tend to uphold narrow patents and invalidate broad ones (Waterson 1990, Cornish 1989, Merges and Nelson 1990).[7]

When the entrant locates at a distance $q_e \leq b$ away from q_p the patent is infringed and the incumbent must decide whether to invoke an infringement trial or not. It is assumed that the filing of an infringement lawsuit by the incumbent is always met with a counterclaim by the accused

[6] The implication of assuming a fencepost patent system is that the probability that infringement is found (given that the entrant has located at $q_e \leq b$ distance away from q_p) does not depend on how close the entrant has located to the incumbent and it is equal to the probability that the validity of the patent will be upheld. Thus, the fencepost patent system implies that the events that the patent is found to be infringed and that the patent is found to be invalid can be treated as mutually exclusive and exhaustive.

[7] Note that, since further entry is not anticipated in our model our analysis and results are not affected by whether the entire patent is invalidated during the infringement/validity trial or only certain claims are found to be invalid (i.e., the patent breadth is narrowed).

infringer that the patent is invalid – a common defense of accused infringers (Cornish 1989, Merges and Nelson 1990). Note that given our assumption of perfect information, the incumbent costlessly identifies infringement as soon as it occurs. This further implies that the incumbent suffers no losses in profits due to infringement and thus the case where the courts award infringement damages to the incumbent is not considered. The legal costs incurred during the infringement trial/validity attack by the incumbent and the entrant are denoted by C_p^T and C_e^T, respectively, and are assumed to be sunk and independent of the breadth of protection and of the entrant's location.[8] Finally, to keep the analysis tractable and the focus on the interplay among the decision to patent, the patent breadth decision and the decision to invoke a trial under infringement, our model does not consider the possibility of settlement or licensing.

The patenting game consists of five stages. In the first stage of the game, the incumbent decides whether to seek patent protection or not. If the incumbent decides not to patent her innovation then the entrant enters at his most preferred location and he and the incumbent compete in prices at the last stage of the game and earn duopoly profits Π_e^{NP} and Π_p^{NP}, respectively. If the incumbent decides to patent her innovation then at the second stage of the game she decides on the patent breadth, b, claimed. In the third stage of the game, a potential entrant observes the incumbent's product and the breadth of protection granted to it and chooses whether or not to enter the market. If the entrant does not enter he earns zero profits while the incumbent operates as a monopolist in the last stage of the game and earns monopoly profits Π_m. If the entrant enters, he does so by choosing the quality q_e of his product relative to that of the incumbent. This decision

[8] By assuming that legal costs are sunk we exclude the possibility of the courts awarding legal fees to either party. In some cases, if infringement is found to be wilful, the court may require that the infringer pays damages up to three times greater than the actual losses due to infringement, opponent's legal costs and court costs (Lerner 1995, Crampes and Langinier 2002). To keep the analysis simple, the possibility of wilful infringement is not examined. Note that, given our assumption of perfect information, the entrant knows in our model whether he has infringed the patent or not (i.e., whether he has located within the incumbent's claims), and thus, the assumption that infringement is not wilful implies that when the entrant infringes the patent he believes that the patent is invalid and thus not infringed.

determines whether the entrant infringes the patent or not, as well as whether the incumbent will invoke a trial in the case the patent is infringed.

If the entrant chooses a quality greater than the patent breadth claimed by the incumbent (i.e., $q_e > b$), then no infringement occurs, and he and the incumbent compete in prices in the last stage of the game and earn duopoly profits Π_e^{NI} and Π_p^{NI}, respectively. If the entrant locates inside the patent breadth claimed by the incumbent (i.e., $q_e \leq b$), the patent is infringed and the incumbent needs to decide whether to invoke a trial or not. This decision is made in the fourth stage of the game. The payoffs for the incumbent and the entrant when the entrant chooses $q_e \leq b$ and the incumbent chooses not to invoke a trial are $(\Pi_p^I)^{NT}$ and $(\Pi_e^I)^{NT}$, respectively. If the incumbent invokes a trial then the validity of the patent is examined. With probability $\mu(b,\alpha)$, the patent is found to be valid (i.e., infringement is found), the entrant is not allowed to market his product and the incumbent operates as a monopolist in the last stage of the game (this follows from our assumption that relocation is prohibitively costly). With probability $1-\mu(b,\alpha)$, the patent is found to be invalid, and the entrant and the incumbent compete in prices. The payoffs for the incumbent and the entrant when the entrant chooses $q_e \leq b$ and the incumbent invokes a trial are $E(\Pi_p^I)^T$ and $E(\Pi_e^I)^T$, respectively. Figure 1 illustrates the extensive form of the game outlined above. The solution to this game is found by backward induction.

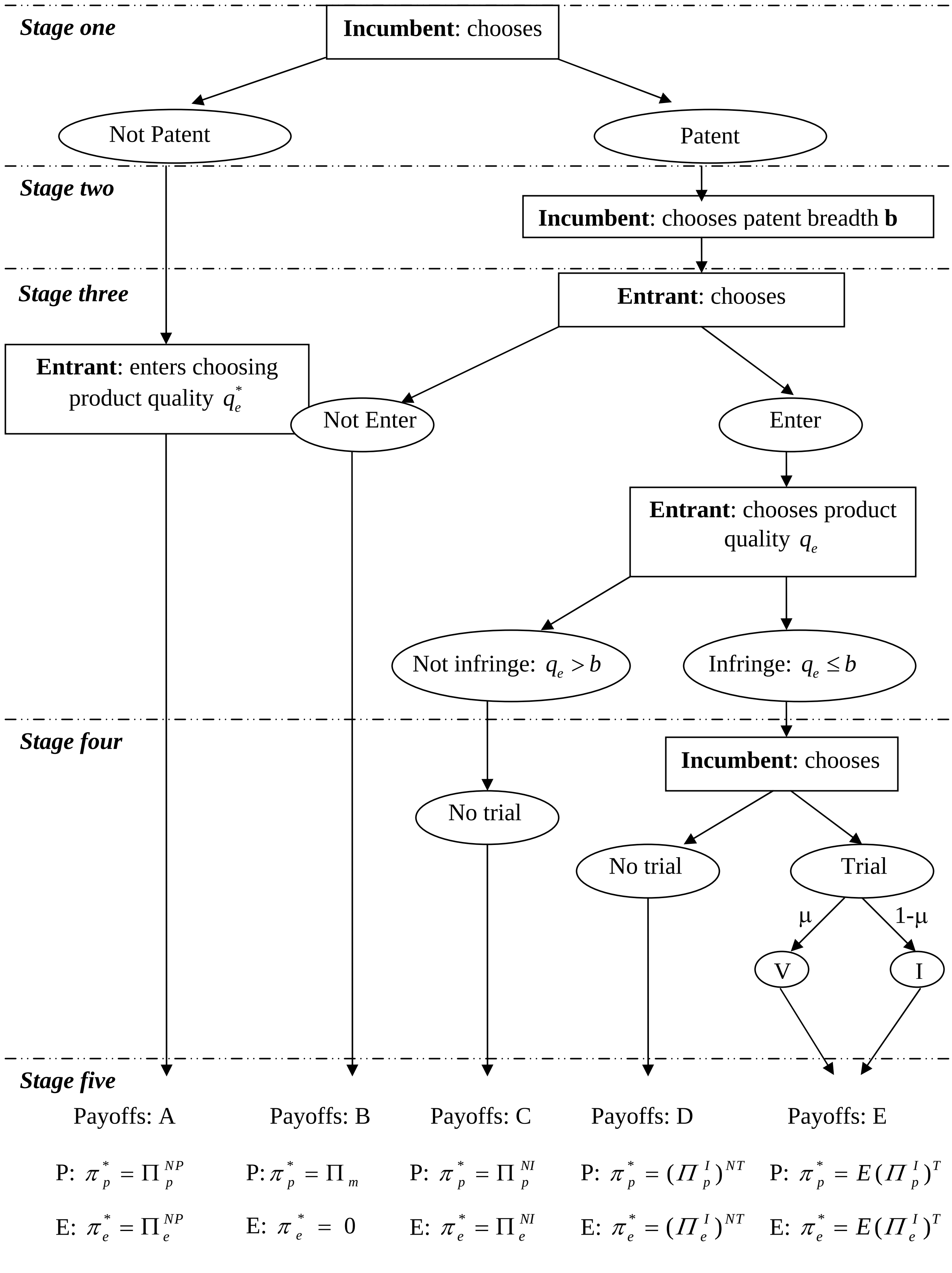

Figure 1. The patenting game

3. The solution of the patenting game

3.1 Stage 5 – The pricing stage

In the fifth stage of the game, when the entrant enters the market, the two players choose their respective prices to maximize their profits. The entrant has the higher quality product and is able to charge the higher price. Profits for both the incumbent and the entrant are increasing in the quality chosen by the entrant, q_e, since the greater is the difference in quality between the two products, the less intense is competition at the final stage of the game.[9] The incumbent's and the entrant's profits at the pricing stage of the game are denoted by $\pi_p(q_e)$ and $\pi_e(q_e)$, respectively, where $\pi_p{}'(q_e) > 0$, $\pi_p{}''(q_e) \geq 0$, $\pi_e{}'(q_e) > 0$ and $\pi_e{}''(q_e) \geq 0$ (see the Appendix for an application).

3.2 Stage 4 – The incumbent's trial decision

As illustrated in Figure 1, under patenting the entrant's quality choice, q_e, will determine whether the patent will be infringed and whether in the case of infringement a trial will take place. When the entrant infringes the patent, the incumbent needs to decide whether to invoke an infringement trial or not. Given the quality chosen by the entrant, the incumbent will invoke a trial when the patent is infringed as long as her expected profits when a trial takes place, $E(\Pi_p^I)^T$, are greater than her profits when a trial does not take place, $(\Pi_p^I)^{NT}$, i.e., $E(\Pi_p^I)^T > (\Pi_p^I)^{NT}$. When the incumbent invokes a trial her expected profits are given by:

(1) $$E(\Pi_p^I)^T = \mu(b,\alpha)\Pi_m + [1-\mu(b,\alpha)]\pi_p(q_e) - C_p^T$$

Equation (1) demonstrates that at trial infringement will be found (or equivalently the validity of the patent will be upheld) with probability $\mu(b,\alpha)$, the entrant will not be allowed in the market and the incumbent will have a monopoly position. Conversely, with probability $1-\mu(b,\alpha)$,

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[9] This is a well-established result in the vertical product differentiation literature; when competitors first choose their locations in the product space and then compete in prices they choose maximum differentiation to relax competition in the pricing stage that would curtail their profits (Lane 1980, Shaked and Sutton 1982, Motta 1993).

infringement will not be found, the entrant will be allowed to market his product and the incumbent and the entrant will operate as duopolists.

When the incumbent does not invoke a trial her profits are given by:

(2) $$(\Pi_p^I)^{NT} = \pi_p(q_e)$$

Equation (2) shows that when the incumbent does not invoke a trial when infringement occurs she shares the market with the entrant and realizes duopoly profits which depend on the entrant's choice of location in the quality product space.

Given the above the incumbent will invoke a trial when her patent is infringed if:

(3) $$E(\Pi_p^I)^T > (\Pi_p^I)^{NT} \Rightarrow \pi_p(q_e) < \Pi_m - \frac{C_p^T}{\mu(b,\alpha)}$$

Equation (3) shows that the incumbent's decision on whether to invoke a trial when her patent is infringed is affected by the entrant's location decision. We denote the quality that makes the incumbent indifferent between invoking and not invoking a trial by $\vec{q}_e \in (0,1)$, i.e.,

$\pi_p(\vec{q}_e) = \Pi_m - \frac{C_p^T}{\mu(b,\alpha)}$, and assume that when the incumbent is indifferent she will choose to not invoke a trial. Since infringement occurs when $q_e \le b$, $\vec{q}_e$ is defined for patent breadth values such that $\vec{q}_e \le b$ and is decreasing in patent breadth at an increasing rate, i.e., $\frac{\partial \vec{q}_e}{\partial b} < 0, \frac{\partial^2 \vec{q}_e}{\partial b^2} < 0$ (see Appendix A_1 for a proof). Thus, the greater is the patent breadth chosen, the smaller is the quality chosen by the entrant that will infringe the patent without invoking a trial.

***Definition 1**. Let $\vec{b} \in (0,1)$ be the patent breadth that is equal to the maximum quality that, when chosen by the entrant, infringes the patent and makes the incumbent indifferent between invoking and not invoking a trial, i.e.,* $\vec{b} = \vec{q}_e \max$.

Given definition 1, $\vec{b} \in (0,1)$ satisfies the condition $\vec{q}_e = b$ and $\vec{q}_e \in (0,1)$ is defined for patent breadth values in the interval $b \in [\vec{b}, 1]$. Figure 2 below illustrates the relationship between the quality chosen by the entrant, q_e, and the incumbent's decision to invoke a trial for any given patent breadth choice, b.

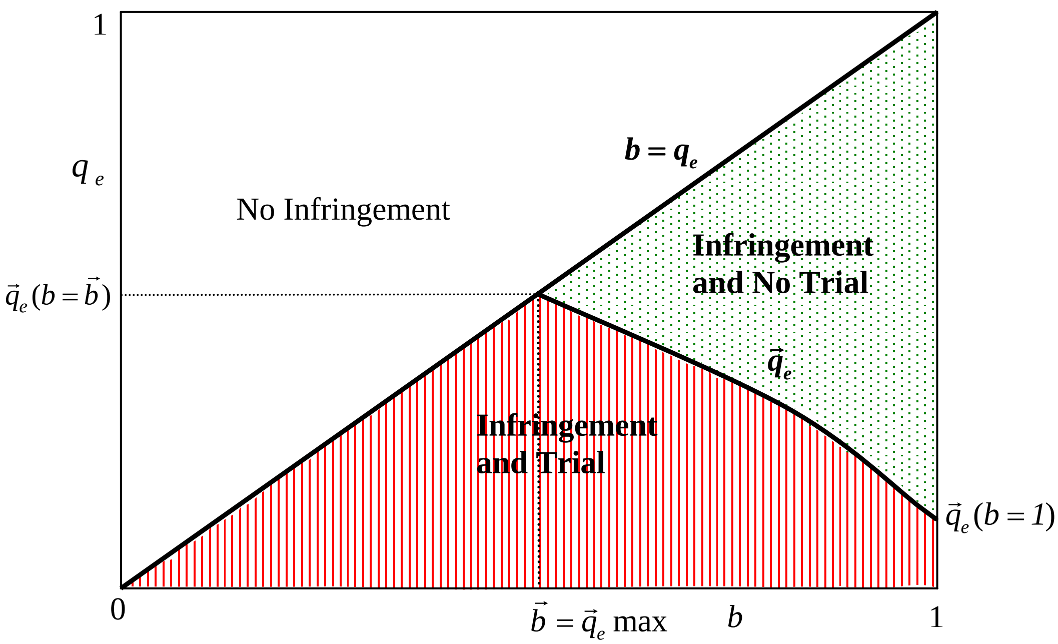

Figure 2. The incumbent's trial decision

As depicted in Figure 2, as long as the entrant chooses a product quality $q_e > b$ the patent is not infringed. When the entrant chooses a product quality q_e such that $\vec{q}_e \leq q_e \leq b$ (i.e., a quality above and to the right of locus $\vec{q}_e$ and below the locus $b = q_e$) the patent will be infringed but the incumbent will not invoke a trial. This outcome is depicted by the dotted area in Figure 2. When the entrant chooses a product quality q_e such that $q_e \leq b$ and $q_e < \vec{q}_e$ (i.e., a quality to the left of locus $\vec{q}_e$ and below the locus $b = q_e$) the patent will be infringed and the incumbent will invoke a trial. This outcome is depicted by the horizontally hatched area in Figure 2. Given the definition of $\vec{q}_e$, as the monopoly profits that can be earned by the incumbent increase, the locus $\vec{q}_e$ shifts upward and the more likely it becomes that a trial will take place under infringement (the infringement and trial area becomes larger). As the incumbent's trial costs and the effect of patent breadth on patent validity increase, the locus $\vec{q}_e$ shifts downward and the less likely it is that the incumbent will find it optimal to invoke a trial under infringement (the infringement and trial area becomes smaller).

3.3 Stage 3 – The entrant's location decision

As illustrated in Figure 1, two cases must be considered regarding the entrant's location decision depending on whether the incumbent has patented her innovation or not. The latter case is considered first.

3.3.1 No patent protection

Given our assumption of possible and costless reverse engineering, the entrant cannot be deterred from entering the market under no patent protection; at the very least, the entrant can locate at $q_e = q_p = 0$, share the market with the incumbent and realize zero profits.

Let q_e^* be the optimal quality the entrant chooses under no patent protection, where q_e^* solves the following problem:

(4) $$\max_{q_e} \Pi_e = \pi_e(q_e) - F_e(q_e) .$$

The first order condition (F.O.C) for the choice of the optimal quality q_e^* is

(5) $$\pi_e{}'(q_e^*) = F_e{}'(q_e^*) .$$

The relationship between the entrant's most preferred location, q_e^*, and patent breadth $\vec{b}$ determines the incumbent's optimal patenting strategy as discussed in the following proposition and corollary.

Proposition 1. *When* $q_e^* > \vec{b}$ *the incumbent will never find it optimal to seek patent protection since she cannot use patent breadth to influence the entrant's location decision.*

Proof. When $q_e^* > \vec{b}$ the entrant will always choose to locate at her most preferred location, q_e^*, regardless of the patent breadth chosen by the incumbent since for patent breadth values smaller than q_e^* the patent is not infringed while for patent breadth values larger than q_e^* the patent is infringed but the incumbent will not invoke a trial (see Figure A_2.1 in Appendix A_2). Knowing that she won't be able to enforce/defend her patent rights, the incumbent will not seek patent protection. Thus, for positive patenting costs, when $q_e^* > \vec{b}$ a patent will not be sought by the incumbent.

Corollary 1. *The greater the entrant's R&D effectiveness, the greater the effect of patent breadth on patent validity,* α *, and the greater the incumbent's trial costs,* C_p^T *, and the smaller the monopoly profits,* Π_m *, the more likely it is that the incumbent will not find it optimal to seek patent protection.*

The results in corollary 1 could be seen in see Figure A_2.1 (in Appendix A_2) where an increase in C_p^T and a decrease in Π_m and $\mu(b,a)$ would shift $\vec{b}$ to the left, making it more likely that the inequality $q_e^* > \vec{b}$ will hold. Also, the greater is the entrant's R&D effectiveness (i.e., the smaller are the entrant's R&D costs), the further away from the incumbent the entrant finds it optimal to locate (the greater is q_e^*) and the more likely it is that the inequality $q_e^* > \vec{b}$ will hold.

3.3.2 Patent protection ($q_e^* \le \vec{b}$ *)*

Given the result in proposition 1, a necessary condition for patent protection to be an optimal strategy for the incumbent is that $q_e^* \le \vec{b}$. Under patent protection and anticipating the incumbent's behavior concerning trial given q_e , the entrant must choose one of four options: (1) Not Enter; (2)

Enter and Not Infringe the Patent; (3) Enter, Infringe the Patent and Induce a Trial; or (4) Enter, Infringe the Patent and Not Induce a Trial. For any given patent breadth, b, the entrant will choose the option that generates the greatest profit.

The outcome of the Not Enter option is straightforward – the entrant earns zero profits. The outcomes of the other three options depend on a number of factors, including patent breadth, R&D costs and trial costs. The benefits and costs associated with the Enter and Not Infringe option are examined first, followed by an examination of the benefits and costs associated with the Enter and Infringe option. The examination of the Enter and Infringe option consists of the examination of the Enter, Infringe and Not Induce a Trial and the Enter, Infringe and Induce a Trial options. Once the net benefits of each option are formulated, the most desirable option for the entrant is determined for any given patent breadth.

3.3.2.1 Entry with no infringement ($q_e > b$)

When the entrant wishes to enter without infringing the patent, he must choose a quality that is greater than the patent breadth, i.e., $q_e > b$. Patent breadth will only be binding as long as $q_e^* \leq b$ since when $q_e^* > b$ the entrant can choose his optimal quality without fear of infringement. Thus, when the entrant wishes to enter and not infringe the patent he will choose his quality q_e^{NI} as follows:

(6) $$q_e^{NI} = \begin{cases} q_e^* & if \quad q_e^* > b \\ b+e & if \quad q_e^* \leq b \end{cases} \quad where \quad e \to 0$$

This quality choice yields profits of:

(7) $$\Pi_e^{NI} = \begin{cases} \pi_e(q_e^*) & if \quad q_e^* > b \\ \pi_e(b+e) - F_e(b+e) & if \quad q_e^* \leq b \end{cases}$$

Result 1. *When the entrant faces a binding patent breadth and he wishes to not infringe the patent, (i.e., $q_e^* \le b$), the entrant's profits under entry and no infringement are decreasing in patent breadth,* b *, at an increasing rate for all* $q_e > q_e^*$ *, i.e.,* $\frac{\partial \Pi_e^{NI}}{\partial b} < 0$, $\frac{\partial^2 \Pi_e^{NI}}{\partial b^2} < 0$.

Since profits are optimal for the entrant under q_e^*, an increase in quality beyond q_e^* results in a reduction in profits.

3.3.2.2 Entry with infringement ($q_e \le b$ *)*

When the entrant decides to enter and infringe the patent he must determine whether to induce the incumbent to invoke a trial or not. The entrant's optimal strategy depends on which of the above two options generates greater profits. The entrant's profits under infringement and trial are determined below followed by an examination of the entrant's profits under infringement and no trial.

The Entrant's Profits under Infringement and Trial

Recall that during an infringement trial there is a probability $\mu(b,\alpha)$ that infringement will be found (i.e., the validity of the patent will be upheld) and a probability $1-\mu(b,\alpha)$ that infringement will not be found (i.e., the patent will be revoked). If infringement is found during trial, the entrant is not allowed to market his product and the patentee earns monopoly profits. If infringement is not found during trial, the entrant is allowed to market his product and the patentee and the entrant operate as duopolists. The optimal quality chosen by the entrant under infringement and trial is determined by solving:

(8) $$\max_{q_e} E\left(\Pi_e^I\right)^T = [1-\mu(b,\alpha)]\cdot \pi_e(q_e) - F_e(q_e) - C_e^T .$$

The F.O.C. for the choice of the optimal quality under infringement and trial $(q_e^I)^T$ is given by:

(9) $$[1-\mu(b,\alpha)]\pi_e'(q_e) = F_e'(q_e)$$

Result 2. *When the entrant infringes the patent knowing that he will face an infringement trial, he chooses a quality* $(q_e^I)^T$ *that is smaller than his most preferred quality, i.e.,* $(q_e^I)^T < q_e^*$*, and is increasing in the patent breadth chosen by the incumbent, i.e.,* $\frac{\partial (q_e^I)^T}{\partial b} > 0$.

The first part of result 2 follows from a comparison of equations (5) and (9) and the intuition behind it is that under infringement and trial there is uncertainty as to whether the entrant will be able to continue in the market and the entrant 'underlocates' to reduce the R&D costs, which are incurred with certainty. The intuition behind the second part of result 2 is that an increase in patent breadth increases the likelihood of the patent being invalidated during an infringement trial (in which case the entrant is allowed to continue in the market) and results in the entrant being willing to invest more in quality (for a proof of the second part of result 2 see Appendix A_3).

Result 3. *The entrant's profits under infringement and trial are increasing in patent breadth,* b*, at an increasing rate, i.e.,* $\frac{\partial E(\Pi_e^I)^T}{\partial b} > 0$, $\frac{\partial^2 E(\Pi_e^I)^T}{\partial b^2} > 0$.

The intuition behind result 3 is that the greater is patent breadth , b, the greater is the probability that infringement will not be found at trial (i.e., that the patent will be found invalid and will be revoked) and the entrant will be allowed in the market (for a formal proof of result 3 see Appendix A_4).

<u>The Entrant's Profits under Infringement and No Trial</u>

This case considers the situation where the choice of the entrant's most preferred quality q_e^* results in patent infringement and trial and the entrant wishes to infringe but not induce a trial. In this case, which can occur only for patent breadth values $b \in [\vec{b}, 1]$ (see Figure 2), the entrant would maximize profits by choosing a quality $(q_e^I)^{NT}$ that is the closest possible to his most preferred quality q_e^* and ensures that the incumbent does not invoke a trial. Thus, to maximize his profits under the infringement and no trial outcome, the entrant will choose the quality $(q_e^I)^{NT} = \vec{q}_e$ (we assume that when the incumbent is indifferent between invoking and not invoking a trial she will choose to not invoke a trial). Note that the quality $(q_e^I)^{NT} = \vec{q}_e$ maximizes the entrant's profits under infringement and no trial when $q_e^* \le \vec{q}_e$. If the loci q_e^* and $\vec{q}_e$ cross for $b \in [\vec{b}, 1]$, there exists a patent breadth $\bar{b} \in (\vec{b}, 1]$ such that $\bar{b} : q_e^* = \vec{q}_e$ and for patent breadth values $b \in [\bar{b}, 1]$ the optimal quality chosen by the entrant under infringement and no trial is given by $(q_e^I)^{NT} = q_e^*$, since the incumbent does not find it optimal to invoke a trial (see Figure 3 below).

Given the above, the entrant's profits under infringement and no trial are given by:

(10) $$(\Pi_e^I)^{NT} = \begin{cases} \pi_e(q_e^*) & if \quad q_e^* > \vec{q}_e \\ \pi_e(\vec{q}_e) - F_e(\vec{q}_e) & if \quad q_e^* \le \vec{q}_e \end{cases}$$

Result 4. *The entrant's profits under infringement and no trial when* $q_e^* \le \vec{q}_e$ *are increasing in patent breadth,* b *, at a decreasing rate, i.e.,* $\dfrac{\partial (\Pi_e^I)^{NT}}{\partial b} > 0$, $\dfrac{\partial^2 (\Pi_e^I)^{NT}}{\partial b^2} < 0$.

The intuition behind result 4 is as follows. Since $\vec{q}_e \ge q_e^*$ and $\dfrac{\partial \vec{q}_e}{\partial b} < 0$ while q_e^* is independent of patent breadth, as patent breadth , b , increases, $\vec{q}_e$ becomes smaller and the closer it is to the optimal location q_e^*. Thus, as $b \in [\vec{b}, 1]$ increases, the closer to his most preferred location, q_e^*, the entrant can locate without inducing the patentee to invoke a trial, and the greater are the entrant's profits under infringement and no trial.

Figure 3 depicts the entrant's quality choices and his profits under patent protection and under no infringement, infringement and trial and infringement and no trial. As this figure shows, the entrant's optimal quality choice depends on the patent breadth chosen by the incumbent. Thus, as long as the incumbent chooses a patent breadth that is not binding (i.e., a $b \in (0, b_0]$ where $b_0 \in (0, \vec{b})$ and $b_0 = q_e^*$) or a patent breadth for which the optimal strategy is to not invoke a trial under infringement (i.e., a $b \in [\bar{b}, 1]$), the entrant will always find it optimal to enter the market and locate at his most preferred location, q_e^*, without invoking a trial.[10] When the patent breadth chosen is such that $b \in (b_0, \vec{b})$, the entrant cannot locate at his most preferred location, q_e^*, without infringing the patent while the incumbent will always find it profitable to invoke a trial when the patent is infringed. In this case, the entrant will have to decide whether to enter and if entry occurs whether to infringe or not infringe the patent knowing that if he infringes a trial will always take place. Finally, when the patent breadth chosen is such that $b \in [\vec{b}, \bar{b})$, the entrant cannot locate at his most preferred location, q_e^*, without infringing the patent but he can, by his choice of location, q_e, in the quality product space affect whether the incumbent will invoke a trial or not when the patent is infringed.

For the profit curves depicted in Figure 3, if the incumbent chooses patent breadth b_1 the entrant will find it optimal to choose the product quality, $(q_e)^{NI} = b_1 + e$ that does not infringe the patent; if the incumbent chooses patent breadth b_2 the entrant will find it optimal to choose the product quality $(q_e^I)^T = f(b_2, q_e^*) < q_e^*$, that infringes the patent and induces the incumbent to invoke a trial while if the incumbent chooses patent breadth b_3 the entrant will find it optimal to choose the product quality $(q_e^I)^{NT} = \vec{q}_e(b_3)$, that infringes the patent and induces the incumbent to not invoke a trial.

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[10] While the patent breadth $b_0 \in (0, \vec{b})$ always exists when $q_e^* \leq b$, the patent breadth $\bar{b} \in (\vec{b}, 1]$ exists only when the loci q_e^* and $\vec{q}_e$ cross. Note that, although Figure 3 depicts the case where $\bar{b}$ exists, as will become evident below, the existence of $\bar{b}$ is not necessary for our results.

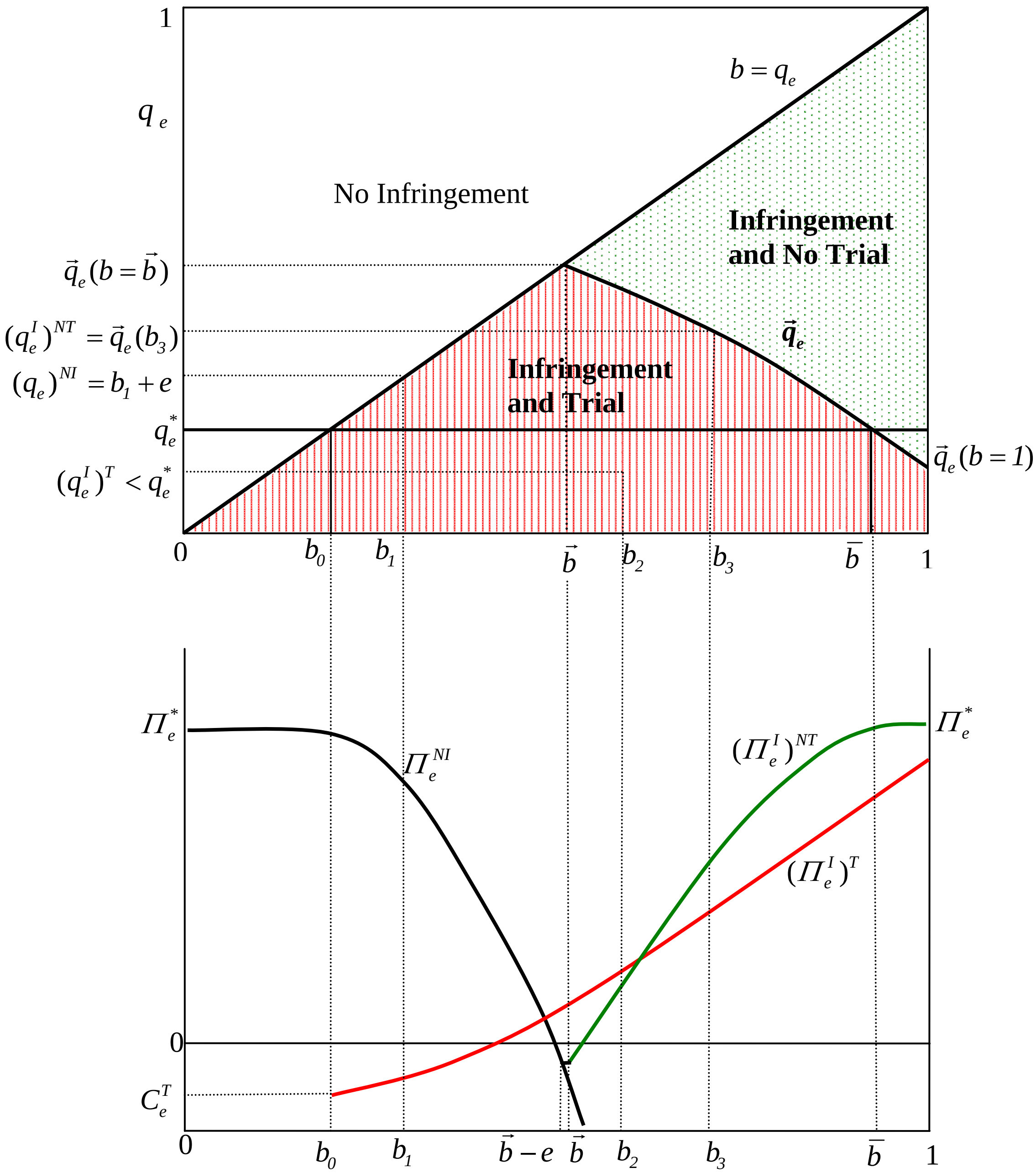

Figure 3. The entrant's quality choices and profits for $q_e^* \leq \vec{b}$ and under no infringement, infringement and trial and infringement and no trial.

Note that the entrant's profits under no infringement when the incumbent chooses the patent breadth $b = \vec{b} - e$ and the entrant (to not infringe) chooses $q_e^{NI} = (\vec{b} - e) + e = \vec{b}$ are equal to his profits under infringement and no trial when the incumbent chooses patent breadth $b = \vec{b}$ and the entrant chooses $(q_e^I)^{NT} = \vec{q}_e = \vec{b}$, i.e., $\Pi_e^{NI}(\vec{b} - e) = (\Pi_e^I)^{NT}(\vec{b}) = \pi_e(\vec{b}) - F_e(\vec{b})$.

Result 5. *When a patent breadth $b \in [\vec{b}, 1]$ is chosen by the incumbent, the entrant will never choose to not infringe the patent since the non infringement strategy is always dominated by the infringement and no trial strategy.*

The intuition behind result 5 is straightforward. As shown in result 1 and result 4, the entrant's profits under no infringement are decreasing in patent breadth while his profits under infringement and no trial are increasing in patent breadth ($\frac{\partial \Pi_e^{NI}}{\partial b} < 0$ and $\frac{\partial (\Pi_e^I)^{NT}}{\partial b} > 0$), respectively. Since $\Pi_e^{NI}(\vec{b} - e) = (\Pi_e^I)^{NT}(\vec{b})$ for any $b \in [\vec{b}, 1]$, it follows that $\Pi_e^{NI}(b) < (\Pi_e^I)^{NT}(b)$.

The entry/infringement decision

Given that the entrant's quality choice depends on the incumbent's patent breadth decision, before we are able to determine the entrant's optimal strategy we must first examine whether there exist some critical patent breadth values that when chosen by the incumbent make the entrant indifferent between the alternative strategies that are available to him.

Definition 2. *Define, $\tilde{b}$, as the patent breadth that makes the entrant indifferent between not infringing the patent and infringing the patent and inducing a trial – i.e., $\tilde{b}$ solves $\Pi_e^{NI}(\tilde{b}) = E(\Pi_e^I)^T(\tilde{b})$ where $\tilde{b} \in (b_0, \bar{b}]$ when $\bar{b} \in (\vec{b}, 1]$ exists and $\tilde{b} \in (b_0, 1]$ when $\bar{b} \in (\vec{b}, 1]$ does not exist.*

Definition 3. *Define, $\tilde{\tilde{b}}$, as the patent breadth that makes the entrant indifferent between infringing the patent and inducing a trial and infringing the patent and not inducing a trial – i.e., $\tilde{\tilde{b}}$ solves*

$E(\Pi_e^I)^T(\tilde{\tilde{b}}) = (\Pi_e^I)^{NT}(\tilde{\tilde{b}})$ *where* $\tilde{\tilde{b}} \in (\vec{b}, \bar{b})$ *when* $\bar{b} \in (\vec{b}, 1]$ *exists and* $\tilde{\tilde{b}} \in (\vec{b}, 1]$ *when* $\bar{b} \in (\vec{b}, 1]$ *does not exist.*

We assume that when the entrant is indifferent between no infringement and infringement and trial he chooses to not infringe the patent while when he is indifferent between infringement and trial and infringement and no trial he choose to infringe and not induce a trial.

Scenario A: Entry deterrence

The entrant will not find it profitable to enter the market if there exists a patent breadth value $\hat{b} \in (b_0, 1]$ that when chosen by the incumbent makes the entrant's profits under no infringement, his expected profits under infringement and trial and his profits under infringement and no trial less than or equal to zero, i.e., $\Pi_e^{NI}(\hat{b}) \leq 0 \wedge E(\Pi_e^I)^T(\hat{b}) \leq 0 \wedge (\Pi_e^I)^{NT}(\hat{b}) \leq 0$.

Proposition 2. *A patent breadth,* $\hat{b}$ *, that deters entry always exists when* $(\Pi_e^I)^{NT}(\vec{b}) \leq 0$ *at* $\vec{b}$ *and* $\Pi_e^{NI}(\tilde{b}) = E(\Pi_e^I)^T(\tilde{b}) < 0$ *at* $\tilde{b}$.

Proof. See Appendix A_5.

Figure 4 illustrates the entry deterrence case where a patent breadth value $\hat{b} \in (b_0, 1]$ (or a range of patent breadth values) exists, that, if chosen by the incumbent, can deter entry.

Result 6. *When the entry deterrence conditions are satisfied and a* $\bar{b} \in (\vec{b}, 1]$ *exists, the patent breadth that deters entry,* $\hat{b}$ *, will always be smaller than the maximum patent breadth possible, i.e.,* $\hat{b} \in (b_0, \bar{b})$.

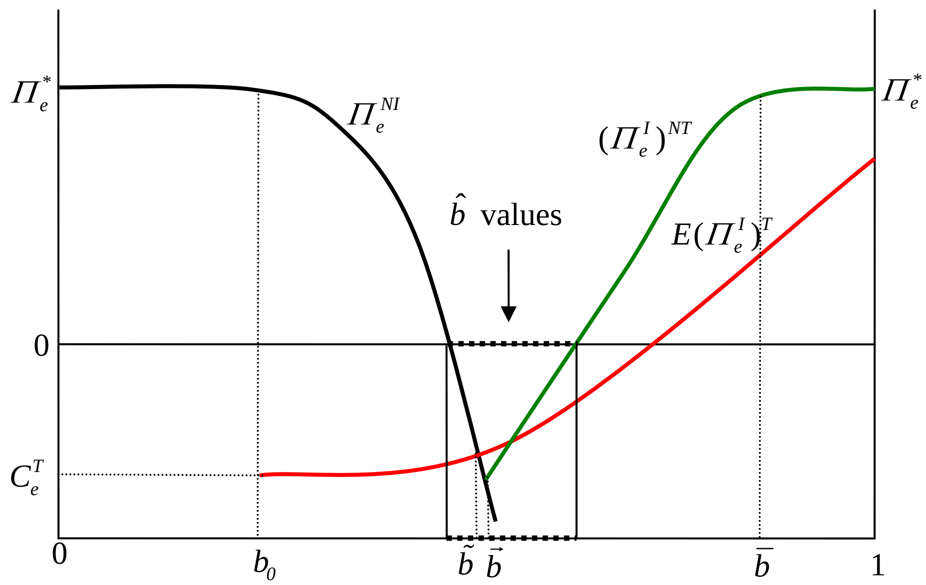

Figure 4. The entrant's profits under no infringement, infringement and trial and infringement and no trial when entry can be deterred.

Scenario B: Entry cannot be deterred

When the entry deterrence conditions in proposition 2 do not obtain, entry cannot be deterred and the optimal strategy for the entrant depends on the relationship between the entrant's profits under no infringement, infringement and no trial and infringement and trial at $\vec{b}$. Two general cases are considered: case I where $(\Pi_e^I)^{NT}(\vec{b}) \geq E(\Pi_e^I)^T(\vec{b})$ and $(\Pi_e^I)^{NT}(\vec{b}) > 0$; and case II where $(\Pi_e^I)^{NT}(\vec{b}) < E(\Pi_e^I)^T(\vec{b})$ and $E(\Pi_e^I)^T(\vec{b}) > 0$. Under each of these two cases, two sub-cases A and B may emerge depending on whether $\bar{b} \in (\vec{b}, 1]$ exists or not, respectively. The four cases, illustrated in Figure 5, exhaust all possible situations where entry cannot be deterred.

- Case I_A

Under this case (Figure 5, panel (i)), the entrant's optimal strategy is to not infringe the patent for patent breadth values $b \in (b_0, \vec{b})$ and to infringe the patent and not induce a trial for patent breadth values $b \in [\vec{b}, 1]$; the strategy of infringing the patent and inducing the incumbent to invoke a trial is never an optimal strategy. This case is most likely to emerge when the entrant's trial costs, C_e^T , are relatively large, and his R&D costs are relatively small making infringement and trial less attractive to the entrant.

- Case I_B

Under this case (Figure 5, panel (ii)), the entrant will find it optimal to not infringe the patent for patent breadth values $b \in (b_0, \vec{b}]$. He will infringe the patent and not induce a trial for patent breadth values $b \in (\vec{b}, \tilde{\tilde{b}}]$ and he will infringe the patent and induce a trial for patent breadth values $b \in (\tilde{\tilde{b}}, 1]$. This case is most likely to emerge when the entrant's trial costs, C_e^T , are relatively large making infringement and trial an attractive strategy to the entrant only for relatively high patent breadth values which imply a higher probability of patent invalidation at trial.

- Case II_A

Under this case (Figure 5, panel (iii)), the entrant's optimal strategy is to not infringe the patent for patent breadth values $b \in (b_0, \tilde{b})$, to infringe the patent and induce a trial for patent breadth values $b \in [\tilde{b}, \tilde{\tilde{b}})$ and to infringe the patent and not induce a trial for patent breadth values $b \in [\tilde{\tilde{b}}, 1]$. This case is most likely to emerge when the entrant's trial costs, C_e^T , and R&D costs are relatively low making infringement with trial attractive to the entrant for relatively low patent breadth values.

- Case II_B

Under this case (Figure 5, panel (iv)), the entrant will find it optimal to not infringe the patent for patent breadth values $b \in (b_0, \tilde{b}]$ and to infringe the patent inducing a trial for patent breadth values $b \in (\tilde{b}, 1]$; the strategy of infringing the patent and not inducing the incumbent to invoke a trial is never an optimal strategy. This case is more likely to emerge when the incumbent's trial costs, C_p^T , are relatively low since the lower are the incumbent's trial costs, the larger is $\vec{q}_e$ and the less attractive is infringement and no trial to the entrant.

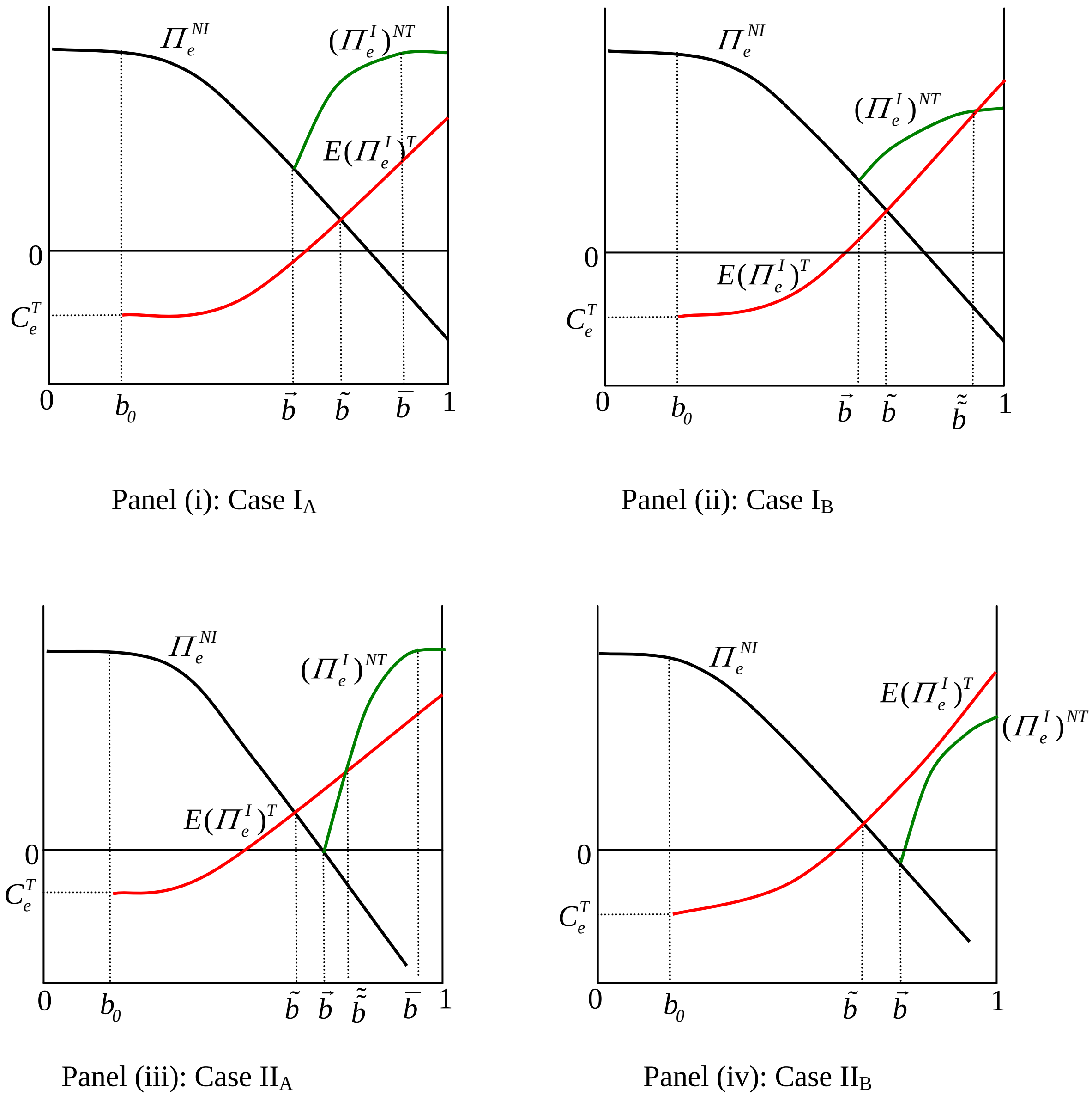

Figure 5. The entrant's profits under no infringement, under infringement and trial and under infringement and no trial when entry cannot be deterred.

3.4 Stage 2 – The patent breadth decision

In stage 2 of the game the incumbent chooses the patent breadth b that maximizes profits, given her knowledge of the entrant's behavior in the third stage of the game. Specifically, the following situations are possible, each one corresponding to one of the scenarios and cases outlined above.

Scenario A: Choose patent breadth to deter entry

If there exists a patent breadth $\hat{b} \in (b_0, 1]$ such that entry can be deterred, then the incumbent will always choose this patent breadth and deter entry. By deterring entry, the incumbent earns monopoly profits, Π_m which are higher than what can be earned under a duopoly.

Scenario B: Entry cannot be deterred (Cases I_A, I_B, II_A and II_B)

The relevant patent breadth values for the incumbent when she wishes to patent the innovation and entry cannot be deterred are such that $b \in (b_0, \bar{b})$.[11]

- Case I_A

Under this case, it is never optimal for the entrant to infringe the patent and induce the incumbent to invoke a trial. The incumbent has to decide whether to choose a patent breadth $b \in (b_0, \vec{b})$ that will induce the entrant not to infringe the patent or to choose a patent breadth $b \in [\vec{b}, \bar{b})$ that will induce the entrant to infringe the patent without inducing a trial.

- Case I_B

Under this case, infringement and trial is optimal for the entrant only for relatively large patent breadth values. In this case, the incumbent has to decide whether to choose a patent breadth $b \in (b_0, \vec{b}]$ and induce the entrant to not infringe the patent, choose a patent breadth $b \in (\vec{b}, \tilde{\tilde{b}}]$ and

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[11] Recall that when $b \in (0, b_0]$ patent breadth is not binding while when $b \in [\bar{b}, 1]$ the incumbent's optimal strategy when the patent is infringed is to not invoke a trial therefore in both cases the entrant locates at his most preferred location, q_e^*. Since q_e^* is where the entrant locates under no patent protection, for positive patenting costs, the patenting strategy is always dominated by the no patenting strategy for these patent breadth values.

induce the entrant to infringe the patent and not force a trial or choose a patent breadth $b \in (\tilde{\tilde{b}}, 1]$ and induce the entrant to infringe the patent and force a trial.

Lemma 1. *The incumbent's profits under infringement and no trial are greater than her profits under infringement and trial for all patent breadth values $b \in (b_0, \bar{b})$; i.e., the infringement and trial strategy is always dominated by the infringement and no trial strategy.*

The incumbent will choose infringement and no trial over infringement and trial when $(\Pi_p^I)^{NT} \geq E(\Pi_p^I)^T$ where $(\Pi_p^I)^{NT}(\vec{q}_e) = \pi_p(\vec{q}_e)$ and

$E(\Pi_p^I)^T((q_e^I)^T) = \mu\Pi_m + (1-\mu)\pi_p((q_e^I)^T) - C_p^T$. Recalling that $\pi_p(\vec{q}_e) = \Pi_m - \dfrac{C_p^T}{\mu(b,\alpha)}$ (from the definition of $\vec{q}_e$), $(\Pi_p^I)^{NT} \geq E(\Pi_p^I)^T$ holds when $\pi_p(\vec{q}_e) \geq \pi_p(q_e^I)^T$. The inequality $\pi_p(\vec{q}_e) \geq \pi_p(q_e^I)^T$ always holds true since $\pi_p{}'(q_e) > 0$ and $\vec{q}_e \geq (q_e^I)^T$ for all patent breadth values that can be chosen by the incumbent when entry cannot be deterred ($b \in (b_0, \bar{b})$); recall that $\vec{q}_e > q_e^*$ and $(q_e^I)^T < q_e^* \; \forall b \in (b_0, \bar{b})$ (see Figure 3). Thus, when deciding between infringement and no trial and infringement and trial, the incumbent's optimal strategy is to always choose infringement and no trial.

Proposition 3. *When entry cannot be deterred and it is either (a) never optimal for the entrant to infringe the patent and face a trial (case I_A) or (b) infringement and trial is optimal for the entrant only for relatively large patent breadth values (case I_B), an optimal strategy for the incumbent is to claim the patent breadth $\vec{b}$ that induces the entrant to infringe the patent and not face a trial.*

Proof: Given the results in lemma 1, the incumbent's decision under cases I_A and I_B is reduced to deciding whether to induce no infringement or infringement and no trial. Since the incumbent's profits are increasing in the entrant's quality choice q_e, both under no infringement and under infringement and no trial i.e., $\Pi_p^{NI} = (\Pi_p^I)^{NT} = \pi_p(q_e)$, the incumbent maximizes her profits by forcing the entrant to locate the furthest away possible in the quality product space. When the incumbent chooses the patent breadth $\vec{b} - e$, the entrant locates the furthest away possible under no infringement by choosing $q_e^{NI} = \vec{b}$; for any $b \geq \vec{b}$ the entrant infringes the patent and does not induce a trial by choosing $(q_e^I)^{NT} = \vec{q}_e$ (see result 5). When the incumbent chooses the patent breadth $\vec{b}$, the entrant infringes the patent and does not induce a trial by choosing $(q_e^I)^{NT} = \vec{b}$.

Since any patent breadth that is greater than $\vec{b}$ (e.g., $\tilde{\tilde{b}}$), will lead to the entrant locating closer to the incumbent (note that $\vec{q}_e = \vec{b}$ at $\vec{b}$ while $\vec{q}_e < b \,\forall\, b \in (\vec{b}, 1]$), the choice of the patent breadth $\vec{b}$ forces the entrant to locate the furthest away possible in the quality product space, maximizing product differentiation and, thus, the incumbent's profits. Given the above, the incumbent is indifferent between the patent breadth values $\vec{b} - e$ and $\vec{b}$ that result in not having the patent infringed and not defending the patent by invoking a trial under infringement, respectively, since both values result in the entrant locating at $\vec{b}$ and in maximum profits for the incumbent, i.e., $\Pi_p^{NI}(\vec{b} - e) = (\Pi_p^I)^{NT}(\vec{b}) = \pi_p(\vec{b})$.

Proposition 3 establishes the main result of the paper, namely that an innovator could find it optimal to patent her innovation even if the patent would not be defended if it were violated. It should be stressed that, even though the incumbent does not find it optimal to defend her patent by claiming the patent breadth $\vec{b}$, the patent is nevertheless valuable. Indeed, without the patent, the entrant would not locate at $\vec{b}$, but rather at $q_e^* < \vec{b}$. The presence of the patent means that the option for the incumbent to take the entrant to court to defend the patent exists. It is the entrant's desire to avoid this option (since doing so increases his profits) that results in him locating further from the incumbent than would be the case in the absence of patent protection. Thus, undefended patents need not signal exploitation or "robbery" by the entrant, but rather represent the outcome of a game in which lack of patent defense emerges as an optimal strategy.

Corollary 3. *When entry cannot be deterred and it is either (a) never optimal for the entrant to infringe the patent and face a trial (case* I_A*) or (b) infringement and trial is optimal for the entrant only for relatively large patent breadth values (case* I_B*), the incumbent maximizes her profits by claiming a relatively narrow rather than broad patent protection as a narrow patent breadth leads to greater product differentiation.*

This result shares a similarity with a result in Yiannaka and Fulton (2006). In their model, where litigation is not endogenous and a trial always occurs upon infringement, if the incumbent finds it optimal to induce non-infringement she chooses a patent breadth that induces the entrant to locate at the edge of patent breadth. In our model the infringement and no trial outcome implies the choice of a patent breadth that similarly induces the entrant to locate by the edge of patent breadth.

The incumbent's profits under case I_A are depicted in Figure 6, panel (i) while her profits under case I_B are depicted in Figure 6, panel (ii).

- Case II_A

Under this case, the incumbent has to decide whether to choose a patent breadth $b \in (b_0, \tilde{b}]$ and induce the entrant to not infringe the patent, choose a patent breadth $b \in (\tilde{b}, \tilde{\tilde{b}})$ and induce the entrant to infringe the patent and face a trial or choose a patent breadth $b \in [\tilde{\tilde{b}}, \bar{b})$ and induce the entrant to infringe the patent and not face a trial.

Lemma 2. *Under case* II_A*, the incumbent's profits under no infringement are greater than her profits under infringement and no trial; i.e., the infringement and no trial strategy is always dominated by the no infringement strategy.*

If the incumbent were to choose to induce non infringement the optimal strategy would be to choose the patent breadth $\tilde{b}$ since this is the patent breadth that forces the entrant to locate the furthest away possible in the quality space without infringing the patent. If the incumbent were to choose to induce infringement and no trial then the optimal strategy would be to choose patent breadth $\tilde{\tilde{b}}$ since this is the patent breadth that induces the entrant to locate the furthest away possible under infringement and no trial (for any $b > \tilde{\tilde{b}}$ the entrant locates closer to the incumbent). Since the incumbent's profits under no infringement and under infringement and no trial are both increasing in the quality chosen by the entrant, q_e, the incumbent is better off choosing $\tilde{b}$ rather than $\tilde{\tilde{b}}$, i.e.,

$\Pi_p^{NI}(\tilde{b}) > (\Pi_p^{I})^{NT}(\tilde{\tilde{b}})$.

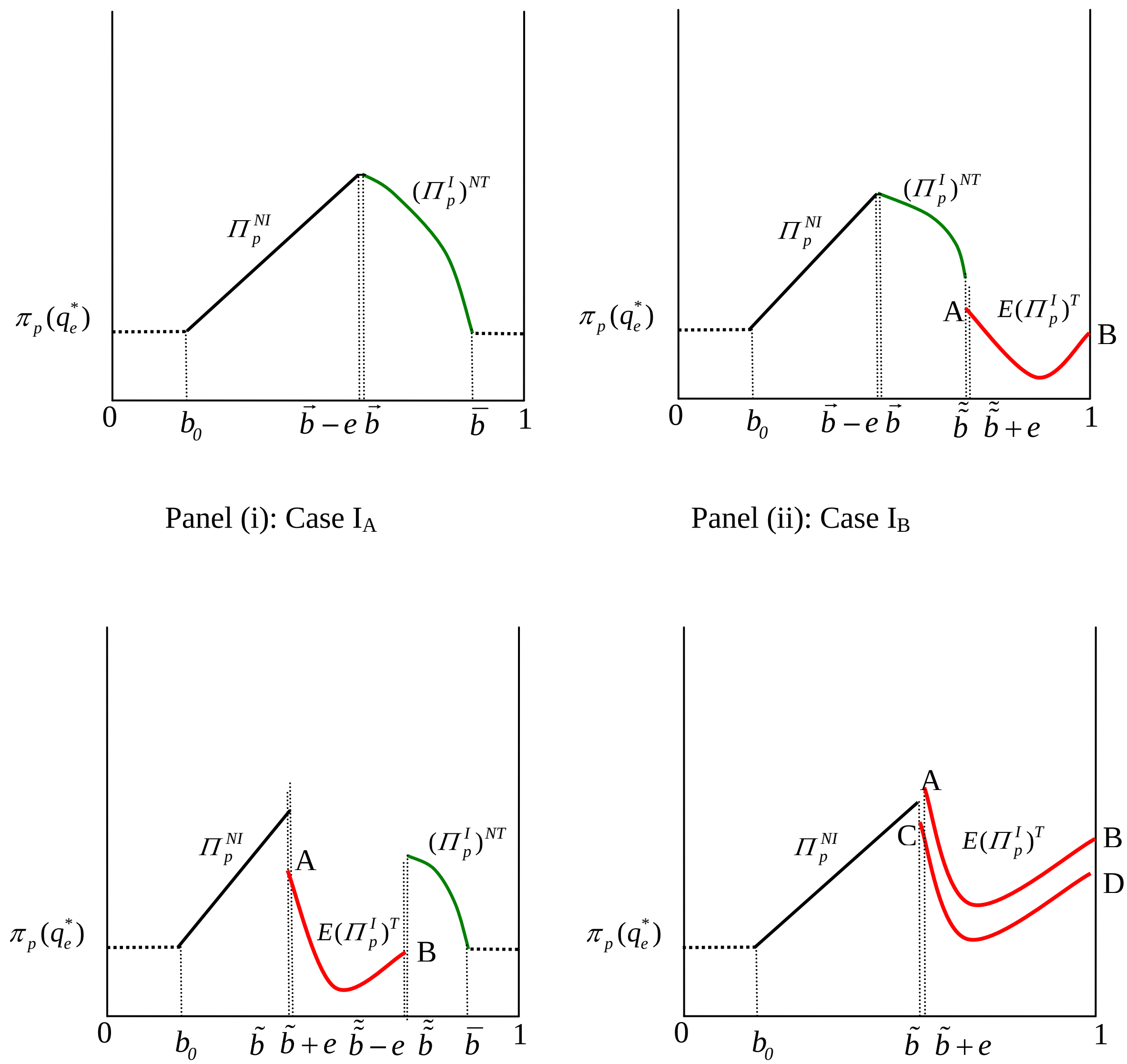
Proposition 4. *When entry cannot be deterred and the entrant finds it optimal to infringe the patent and face a trial for intermediate patent breadth values and infringe the patent and not face a trial for relatively large patent breadth values* (*i.e., case* II_A*), the optimal strategy for the incumbent is to choose a patent breadth value that induces the entrant to not infringe the patent, i.e.,* $b^{NI} = \tilde{b}$.

Proof: The result in proposition 4 follows directly from lemma 1 and 2.

Case II_A when the optimal patent breadth under infringement and trial is given by $b^I = \tilde{b} + e$ is depicted in Figure 6, panel (iii).

- Case II_B

Under this case, it is never optimal for the entrant to infringe the patent without inducing a trial. The incumbent has to decide whether to choose a patent breadth $b \in (b_0, \tilde{b}]$ and induce the entrant to not infringe the patent or to choose a patent breadth $b \in (\tilde{b}, 1]$ and induce the entrant to infringe the patent and induce a trial. This case has been examined by Yiannaka and Fulton (2006) who find that the incumbent will induce non infringement by claiming $b^{NI} = \tilde{b}$ or induce infringement and trial by claiming either $b^I = \tilde{b} + e$ or $b^I = 1$. Case II_B when the optimal patent breadth under infringement and trial is given by $b^I = \tilde{b} + e$ is depicted in Figure 6, panel (iv). As shown in this figure, when the incumbent's expected profits under infringement and trial are given by the curve AB, the incumbent's optimal strategy is to claim patent breadth $b^I = \tilde{b} + e$ and induce infringement and trial while if her expected profits are given by CD, the incumbent's optimal strategy is to claim $b^{NI} = \tilde{b}$ and induce non infringement.



Panel (i): Case I_A

Panel (ii): Case I_B

Panel (iii): Case II_A

Panel (iv): Case II_B

Figure 6. The incumbent's profits when entry cannot be deterred (cases I_A, I_B, II_A and II_B).

3.5 Stage 1 – The patenting decision

In stage 1 of the game the incumbent decides whether to patent her innovation or not given her knowledge of the entrant's response to her patent breadth and trial decisions. The incumbent will choose to patent her innovation when the profits earned under patenting are greater than the profits earned under no patent protection, $\Pi_p^P \geq \Pi_p^{NP}$.

As described in the preceding sections if the incumbent chooses not to patent, entry cannot be deterred; the entrant will enter the market choosing his most preferred quality q_e^* and the incumbent will earn profits $\Pi_p^{NP}(q_e^*)$. As shown in proposition 1, the incumbent will always find it optimal not to patent the innovation when $q_e^* \geq \vec{b}$ regardless of the level of patenting costs. In this case, had the incumbent chosen to patent, the entrant would always choose q_e^* and the incumbent would not find it optimal to invoke a trial. Thus, if under patenting the incumbent can never enforce her patent rights when the patent is infringed she always chooses not to patent.

The incumbent may find it optimal to patent her innovation when the entrant's location choice affects her decision to invoke a trial under infringement (i.e., when $q_e^* < \vec{b}$). In this case, whether the incumbent will find it optimal to patent or not depends on the magnitude of the parameters of the model (the level of monopoly profits, trial and R&D costs and the incumbent's patenting costs).

An application of the general model where the use of specific functional forms that satisfy the general model assumptions facilitates the exposition of the main results is presented in the Appendix.

4. Concluding remarks

A game theoretic model was developed to examine how the innovator's ability to enforce her patent rights affects and is affected by her decision to patent her innovation and her patent breadth decision, and to show that unenforced patents can still be valuable for patent holders. The innovator in our model seeks patent protection for a product innovation under potential entry by a firm producing a better quality product.

Patenting is only optimal if the existence of the patent causes the entrant to locate further away from the incumbent in product space than would be the case in the absence of a patent. The incumbent is unable to use a patent to influence the entrant's location choice when her trial costs are large, when the monopoly profits that can be captured are relatively small and when the entrant's R&D costs are relatively low. Under these conditions, the patenting strategy is always dominated by the no patenting strategy for positive patenting costs.

When these conditions do not obtain, the incumbent is able to use patent breadth to induce the entrant to locate further away in the quality product space than he would have located under no patent protection. When such relocation is possible, patenting may become an optimal strategy, with the patenting choice dependent on the relative magnitude of the patenting costs vis-à-vis the extra profits that are obtained as a result of inducing the entrant to alter his location choice.

An important result of the paper is that a patent can be effective at altering the entrant's location choice (and thus the rents that can be captured by the patent) even if the incumbent innovator does not defend the patent when it is violated. This result occurs because the incumbent's decision to defend a patent by invoking a trial is one that the entrant can influence by his choice of location. Under a specific set of conditions – most notably when the entrant's R&D costs are relatively low and his trial costs are relatively high – the incumbent selects a patent breadth that results in the entrant choosing a location that, even though it infringes the patent and the incumbent does not find it optimal to defend the patent by invoking a trial, it is still advantageous for the incumbent. Thus, situations where the patentee does not actively defend violated patents may in fact be optimal – the entrant is not getting away with robbery, but instead has been induced to select this location by the incumbent. Such a strategy is optimal because a relatively narrow rather than broad patent breadth achieves greater product differentiation and thus greater profits for the incumbent.

The above results hold under our model assumptions of complete and perfect information, single entry, a deterministic R&D process and possible and costless reverse engineering of the innovator's product. Relaxing the above assumptions is the focus of future research.

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Technical Appendix A: General Model

A_1. The effect of patent breadth, b, on $\vec{q}_e$

Totally differentiating $\pi_p(\vec{q}_e) = \Pi_m - \frac{C_p^T}{\mu(b)}$ with respect to b and $\vec{q}_e$ gives:

$$\frac{\partial \pi_p(\vec{q}_e)}{\partial \vec{q}_e} d\vec{q}_e + \frac{\partial \pi_p(\vec{q}_e)}{\partial b} db = \frac{\partial(\Pi_m - \frac{C_p^T}{\mu(b)})}{\partial \vec{q}_e} d\vec{q}_e + \frac{\partial(\Pi_m - \frac{C_p^T}{\mu(b)})}{\partial b} db \Rightarrow \pi_p{}'(\vec{q}_e) d\vec{q}_e = \frac{C_p^T \mu'(b)}{[\mu(b)]^2} db \Rightarrow$$

$\frac{d\vec{q}_e}{db} = \frac{C_p^T \mu'(b)}{[\mu(b)]^2 \pi_p{}'(\vec{q}_e)}$. Given the properties of functions $\mu(b)$ and $\pi_p(q_e)$ it follows that $\frac{d\vec{q}_e}{db} < 0$.

From the above it easily follows that $\frac{d^2\vec{q}_e}{db^2} = \frac{C_p^T \mu''(b)\mu(b) - 2C_p^T [\mu'(b)]^2}{[\mu(b)]^3 \pi_p{}'(\vec{q}_e)} < 0$.

A_2. Depiction of proposition 1

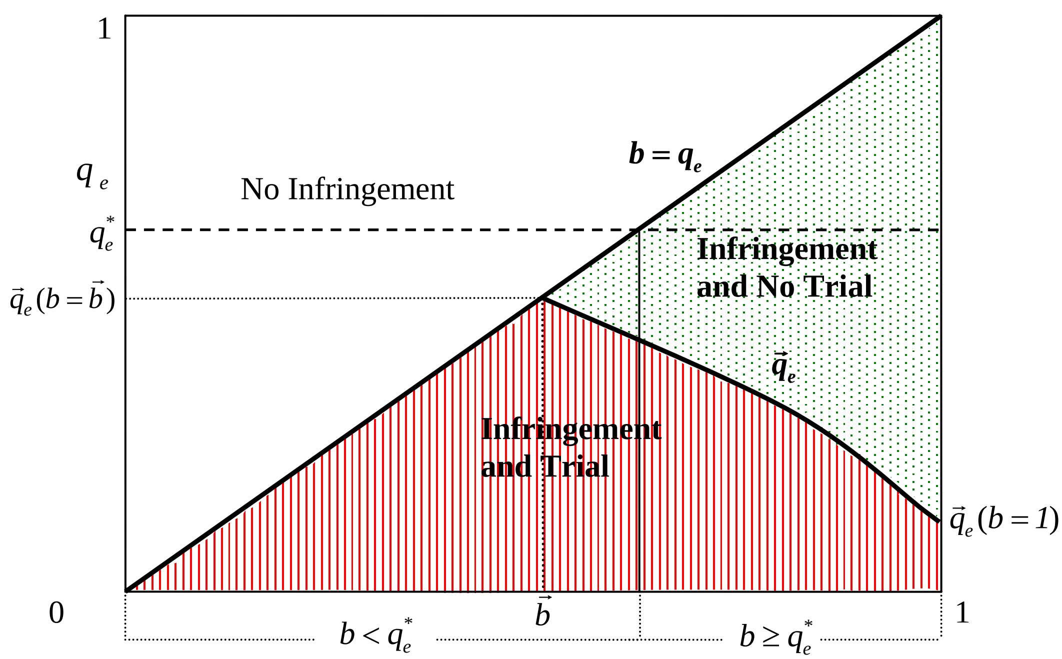

Figure A_2.1. Patenting is not optimal for $q_e^* > \vec{b}$

A_3. The effect of patent breadth, b, on $(q_e^I)^T$

Totally differentiating $[1-\mu(b)]\pi_e'((q_e^I)^T) = F_e'((q_e^I)^T)$ with respect to b and $(q_e^I)^T$ gives:

$$\frac{\partial[1-\mu(b)]\pi_e'((q_e^I)^T)}{\partial(q_e^I)^T} d(q_e^I)^T + \frac{\partial[1-\mu(b)]\pi_e'((q_e^I)^T)}{\partial b} db = \frac{\partial F_e'((q_e^I)^T)}{\partial(q_e^I)^T} d(q_e^I)^T + \frac{\partial F_e'((q_e^I)^T)}{\partial b} db \Rightarrow$$

$$\{[1-\mu(b)]\pi_e''((q_e^I)^T)\}d(q_e^I)^T - \{\mu'(b)\pi_e'((q_e^I)^T)\}db = F_e''((q_e^I)^T)d(q_e^I)^T + 0 \Rightarrow$$

$$\frac{d(q_e^I)^T}{db} = \frac{\mu'(b)\pi_e'((q_e^I)^T)}{[1-\mu(b)]\{\pi_e''((q_e^I)^T) - F_e''((q_e^I)^T)\}} > 0.$$

Note that the denominator in $\frac{d(q_e^I)^T}{db}$ is the second order condition (S.O.C.) for a maximum of equation (8) and it is thus negative.

A_4. The effect of patent breadth, b, on the entrant's expected profits under infringement and trial

The entrant's expected profits under infringement and trial are given by

$E\left(\Pi_e^I\right)^T = [1-\mu(b,\alpha)]\cdot\pi_e((q_e^I)^T) - F_e((q_e^I)^T) - C_e^T$. Differentiating these profits with respect to patent breadth, b, gives: $\frac{\partial E\left(\Pi_e^I\right)^T}{\partial b} = \frac{\partial[1-\mu(b)]}{\partial b}\cdot\pi_e((q_e^I)^T) + [1-\mu(b)]\frac{\partial\pi_e((q_e^I)^T)}{\partial(q_e^I)^T}\frac{\partial(q_e^I)^T}{\partial b} - \frac{\partial F_e((q_e^I)^T)}{\partial(q_e^I)^T}\frac{\partial(q_e^I)^T}{\partial b} =$

$$= -\mu'(b)\pi_e((q_e^I)^T) + \{[1-\mu(b)][\pi_e'((q_e^I)^T) - F_e'((q_e^I)^T)]\}\frac{\partial(q_e^I)^T}{\partial b}.$$

Noticing that the term $\{[1-\mu(b)][\pi_e'((q_e^I)^T) - F_e'((q_e^I)^T)]\} = 0$ due to equation (9) (F.O.C.) it follows that

$$\frac{\partial E\left(\Pi_e^I\right)^T}{\partial b} > 0.$$

Given the above $\frac{\partial^2 E\left(\Pi_e^I\right)^T}{\partial b^2} = -\mu''(b)\pi_e((q_e^I)^T) - \mu'(b)\pi_e'((q_e^I)^T)\frac{\partial(q_e^I)^T}{\partial b} > 0$.

A_5. Conditions under which entry can be deterred – proof of proposition 2.

When either $(\Pi_e^I)^{NT}(\vec{b}) > 0$ at $\vec{b}$ or $E(\Pi_e^I)^T(\tilde{b}) > 0$ at $\tilde{b}$ there is no patent breadth value that, when chosen by the incumbent, can deter the entrant from entering the market. This is so because $\frac{\partial \Pi_e^{NI}}{\partial b} < 0 \;\; \forall b \in (b_0, \vec{b}]$ (result 1), $\frac{\partial (\Pi_e^I)^{NT}}{\partial b} > 0 \;\; \forall b \in [\vec{b}, \bar{b})$ (result 2) and $\frac{\partial E(\Pi_e^I)^T}{\partial b} > 0 \;\; \forall b \in (b_0, 1]$ (result 3) while at $\vec{b}$ $\Pi_e^{NI}(\vec{b}) < (\Pi_e^I)^{NT}(\vec{b})$ and at $\tilde{b}$ $\Pi_e^{NI}(\tilde{b}) = E(\Pi_e^I)^T(\tilde{b})$. Thus, the condition $(\Pi_e^I)^{NT}(\vec{b}) \le 0 \wedge E(\Pi_e^I)^T(\tilde{b}) < 0$ is a necessary and sufficient condition for entry deterrence.

Appendix: An Application

This Appendix provides an application of the general model where the use of specific functional forms that satisfy the general model assumptions facilitates the exposition of the main results.

Assume that in the vertical market described in section 2.1 the incumbent's product, q_p, provides consumers with utility $U_p = V - p_p$, where V is a base level of utility and p_p is the product's price; the entrant's product, $q_e \in (0,1]$, provides consumers with utility $U_e = V + \lambda q_e - p_e$, where λ is a differentiating consumer attribute uniformly distributed with unit density $f(\lambda) = 1$ in the interval $\lambda \in [0,1]$ and p_e is the price of the entrant's product. It is assumed that V is large enough so that $V \ge p_i \;\forall i = p, e$ and $U_i \ge 0$ and $U_i > U_j$ so the market is always served by at least one product. In this market, the consumer who is indifferent between the two products has a λ value denoted by λ^*, where λ^* is determined as:

$$U_p = U_e \Rightarrow \lambda^* = \frac{p_e - p_p}{q_e}.$$

Since each consumer consumes one unit of the product of her choice, the demand for the products produced by the incumbent and the entrant are given by $y_p = \lambda^*$ and $y_e = 1 - \lambda^*$, respectively.

Given the above, in the absence of entry, the incumbent will charge $p_p = V$ and earn monopoly profits $\Pi_m = V - F_p$, in the fifth stage of the patenting game. If entry occurs, the problem facing duopolist i

is to choose price p_i to maximize profit $\pi_i = p_i y_i$ $(i = p, e)$, where $y_p = \frac{p_e - p_p}{q_e}$ and $y_e = \frac{q_e + p_p - p_e}{q_e}$.

Recall that the R&D costs, F_p and F_e for the incumbent and the entrant, respectively, are assumed to be sunk at this stage in the game and thus are not included in the profit expression. The Nash equilibrium in prices and the resulting outputs and profits, are given by:

Incumbent: $p_p^* = \frac{q_e}{3}$, $y_p^* = \frac{1}{3}$, $\pi_p^* = \frac{q_e}{9}$

Entrant: $p_e^* = \frac{2q_e}{3}$, $y_e^* = \frac{2}{3}$, $\pi_e^* = \frac{4q_e}{9}$

As outlined in section 3.1, the entrant has the higher quality product and is able to charge the higher price. Profits for both the incumbent and the entrant are increasing in the quality chosen by the entrant, q_e; thus, maximum product differentiation is desirable to both players.

The formulation $F_e = \beta \frac{q_e^2}{2}$ $(\beta \geq \frac{4}{9})$ is used for the entrant's R&D costs where the restriction on the parameter β ensures that the quality chosen by the entrant, q_e, is bounded between zero and one. Given the relationship between patent breadth and the probability that the patent will be found valid at an infringement trial, μ can be expressed as $\mu(b) = 1 - \alpha b$ where $\alpha \in (0,1)$ is the validity parameter.

Substitution of the appropriate functional forms into equations (1), (2) and (3) yields the quality $\vec{q}_e = 9(\Pi_m - \frac{C_p^T}{1 - ab})$ that makes the incumbent indifferent between invoking and not invoking a trial. It is straightforward to show that a $\vec{q}_e \in [\vec{b}, 1)$ exists when the condition $C_p^T < \Pi_m < \frac{1}{9} + \frac{C_p^T}{1 - \alpha}$ is satisfied.[12]

[12] Note that when $\Pi_m \geq \frac{1}{9} + \frac{C_p^T}{1 - \alpha}$ the locus $\vec{q}_e$ is above the locus $q_e = b$ and invoking a trial when the patent is infringed ($q_e \leq b$) is always an optimal strategy for the incumbent, regardless of the quality chosen by the entrant while when $\Pi_m \leq C_p^T$, $\vec{q}_e$ is below the locus $q_e = b$ and invoking a trial when the patent is infringed ($q_e \leq b$) is never an optimal strategy for the incumbent, regardless of the quality chosen by the entrant.

Substitution of the appropriate functional forms into equations (4) and (5) yields the entrant's most preferred quality choice $q_e^* = \frac{4}{9\beta}$, where the less costly it is to produce the better quality product (i.e., the smaller is β), the further away from the incumbent the entrant locates. Substitution of the appropriate functional forms into equations (8) and (9) yields the entrant's optimal quality choice under infringement and trial, $(q_e^I)^T = \frac{4\alpha b}{9\beta} = \alpha b q_e^*$, which is proportional to the entrant's most preferred quality choice and the incumbent's patent breadth.

Given the above, it is straightforward to show that the condition $q_e^* > \vec{b}$ which implies that the incumbent will never find it optimal to seek patent protection (Proposition 1) is satisfied for R&D cost values, β, such that $\beta < \frac{8\alpha}{9(1 + 9\alpha\Pi_m - \sqrt{1 + 36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2})} = \beta_0$ where $\beta_0 > \frac{4}{9}$

$\forall \alpha \in (0,1), C_p^T \geq 0 \wedge \Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha})$ (for a formal proof see Appendix B_1). The condition $q_e^* \leq \vec{b}$ under which the incumbent may find it profitable to seek patent protection is satisfied for R&D cost values, β, such that $\beta \geq \beta_0$.

Figure A.1 below illustrates the combinations of values of the entrant's R&D effectiveness, β, and the monopoly profits that can be captured by the incumbent, Π_m, for which patent protection will and will not be sought.

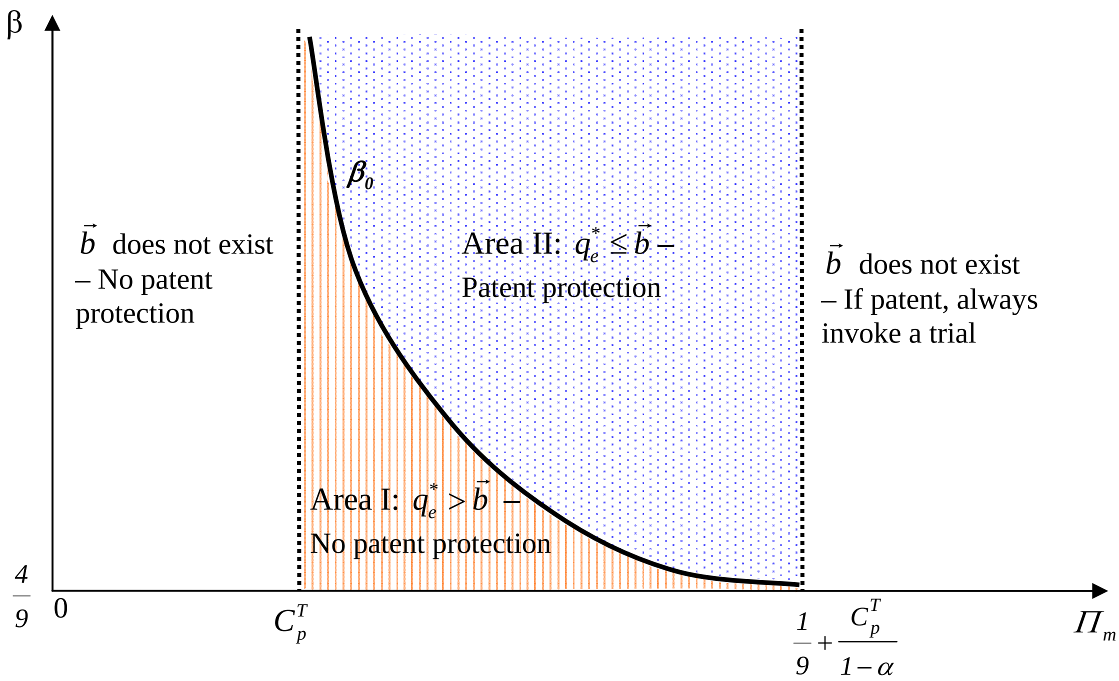

Figure A.1. Combinations of β and Π_m values for which patent protection will and will not be sought.

As depicted in Figure A.1, when $\Pi_m \to C_p^T$, $\beta_0 \to \infty$ while when $\Pi_m \to \frac{1}{9}+\frac{C_p^T}{1-\alpha}$, $\beta_0 \to \frac{4}{9}$ $\forall \alpha \in (0,1)$ and $C_p^T \ge 0$; thus, $\beta_0 > \frac{4}{9}$ $\forall \alpha \in (0,1), C_p^T \ge 0 \wedge \Pi_m \in (C_p^T, \frac{1}{9}+\frac{C_p^T}{1-\alpha})$. For combinations of β and Π_m values to the right of locus C_p^T and below the locus β_0 (Area I in Figure A.1), $\vec{b}$ exists, $q_e^* > \vec{b}$ and the incumbent will not find it optimal to seek patent protection. For combinations of β and Π_m values to the left of locus $\frac{1}{9}+\frac{C_p^T}{1-\alpha}$ and to the right of locus β_0 (Area II in Figure A.1), $\vec{b}$ exists, $q_e^* \le \vec{b}$, the incumbent might find it optimal to seek patent protection and the entrant's location decision will determine whether the incumbent will find it optimal to invoke a trial in the case of infringement. Note that Figure A.1 provides an illustration of corollary 1. For instance, as the validity parameter, α, increases, the locus $\frac{1}{9}+\frac{C_p^T}{1-\alpha}$ shifts to the right, the locus β_0 shifts upwards and the no patent protection area becomes larger.

Figure A.2 depicts the combination of the entrant's R&D effectiveness, β, and trial cost, C_e^T, values (for given values of the exogenous parameters) that give rise to entry deterrence (scenario A) and the four cases that may emerge when entry cannot be deterred (scenario B). The conditions under which scenario A and scenario B emerge are given in Appendix B_2 and B_3, respectively. For definitions of all the loci in Figure A.2 see Appendix B_2 and B_3.

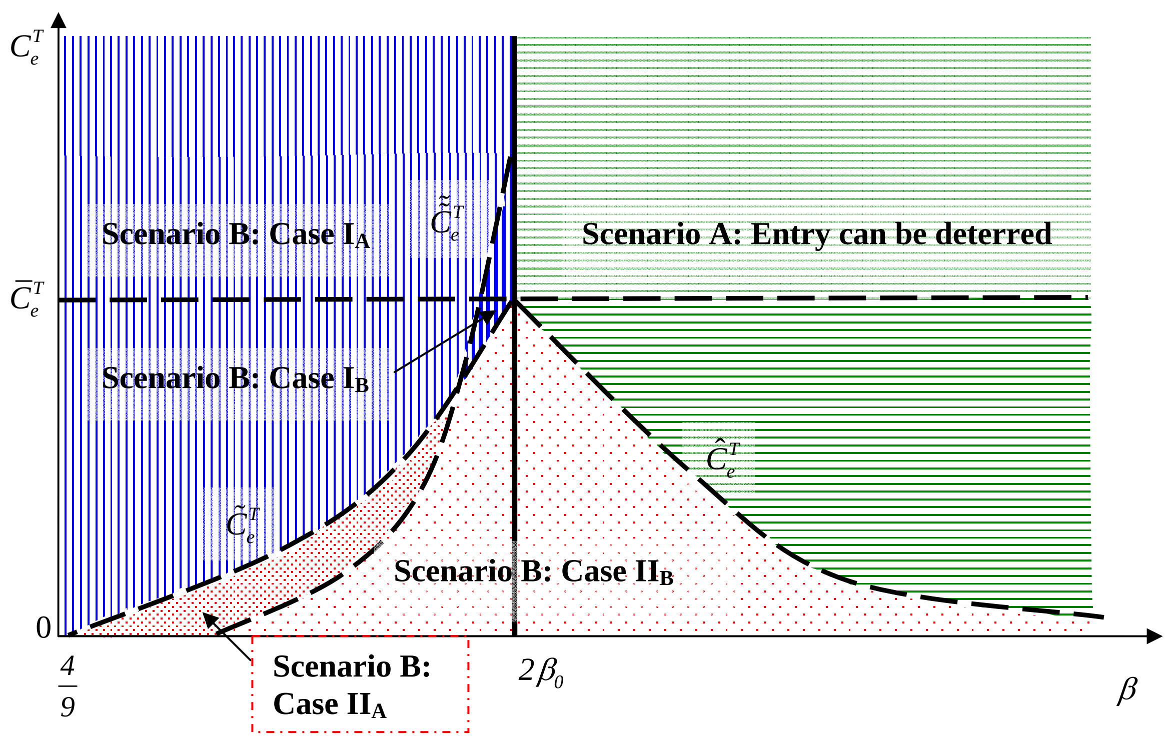

Figure A.2. Combinations of β and C_e^T values for which entry can and cannot be deterred.

The combinations of β and C_e^T values to the right of locus $\beta = 2\beta_0$ and above the locus $\hat{C}_e^T$ (i.e., $\beta \geq 2\beta_0$ and $C_e^T \geq \hat{C}_e^T$), depicted by the horizontally hatched area in Figure A.2, give rise to the entry deterrence outcome. Entry deterrence is possible if both β and C_e^T values are relatively large. The combinations of β and C_e^T values to the left of locus $\beta = 2\beta_0$ and above the loci $\tilde{C}_e^T$ and $\tilde{\tilde{C}}_e^T$, depicted by the lightly vertically hatched area in Figure A.2, correspond to case I_A (Figure 5, panel (i)) while the combinations of β and C_e^T values to the left of locus $\beta = 2\beta_0$, above the locus $\tilde{C}_e^T$ and below the locus $\tilde{\tilde{C}}_e^T$, depicted by the heavily vertically hatched area in Figure A.2, correspond to case I_B (Figure 5, panel (ii)). The combinations of β and C_e^T values above the locus $\tilde{\tilde{C}}_e^T$ and below the locus $\tilde{C}_e^T$, depicted by the heavily dotted area in Figure A.2, correspond to case II_A (Figure 5, panel (iii)). Finally, the combinations of β and C_e^T values to the left of locus $\hat{C}_e^T$ and to the right of loci $\tilde{C}_e^T$ and $\tilde{\tilde{C}}_e^T$, depicted by the lightly dotted area in Figure A.2, correspond to case II_B (Figure 5, panel (iv)).

Table 1 presents the effect of the exogenous parameters on the incumbent's decision to patent when entry can be deterred (Scenario A) and when entry cannot be deterred (Scenario B, cases I_A, I_B, II_A and II_B). The incumbent's profits under no patent protection are given by $\Pi_p^{NP} = \pi(q_e^*) = \frac{4}{81\beta}$ while her profits under patent protection when entry can be deterred (Scenario A) are given by $\Pi_p^P = \Pi_m - z$. When entry cannot be deterred (Scenario B) the incumbent's profits are given by $\Pi_p^P = \frac{\vec{b}}{9} - z$ under cases I_A and I_B and by $\Pi_p^P = \frac{\tilde{b}}{9} - z$ under cases II_A and II_B. Finally, when entry cannot be deterred (Scenario B) and the incumbent finds it optimal to induce infringement and trial as in case II_B her profits are given by

$E(\Pi_p^I)^T = (1 - ab)\Pi_m + a^2 b^2 \frac{q_e^*}{9} - C_p^T$.

Table 1. Effect of exogenous parameters on the incumbent's decision to patent

Scenarios	Patenting profitable when $\Pi_p^P \geq \Pi_p^{NP}$	Impact on the incumbent's decision to patent				
		Π_m	β	α	C_p^T	C_e^T
Scenario A: $b=\hat{b}$	$z \leq \Pi_m - \frac{4}{81\beta}$	(+)	(+)	0	0	0
Scenario B: case I_A, case I_B: $b=\vec{b}$	$z \leq \frac{\vec{b}}{9} - \frac{4}{81\beta}$	(+)	(+)	(−)	(−)	0
Scenario B: case II_A, case II_B: $b=\tilde{b}$	$z \leq \frac{\tilde{b}}{9} - \frac{4}{81\beta}$			(−)	0	(+)
Scenario B: case II_B: $b=\tilde{b}+e$	$z \leq (1-\alpha(\tilde{b}+e))\Pi_m + \frac{4\alpha^2(\tilde{b}+1)^2}{81\beta} - C_p^T$	(+)			(−)	(+)
Scenario B: case II_B: $b=1$	$z \leq (1-\alpha)\Pi_m + \frac{4(\alpha^2-1)}{81\beta} - C_p^T$	(+)	(+)		(−)	

Note: blank cells imply that the impact cannot be determined without knowledge of the magnitude of the parameters

As shown in Table 1, the greater are the monopoly profits that can be captured by the incumbent and the entrant's R&D and trial costs, and the smaller are the validity parameter and the incumbent's trial costs, the more likely it is that patenting will result in greater profits than no patenting for the incumbent.

Technical Appendix B: Application

B_1. Conditions for no patent protection $q_e^* > \vec{b}$

Given that $q_e^* = \frac{4}{9\beta}$ and $\vec{b} = \frac{1+9\alpha\Pi_m - \sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2}}{2\alpha}$ [13] the condition $q_e^* > \vec{b}$ can be written as $\beta < \frac{8\alpha}{9(1+9\alpha\Pi_m - \sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2})} = \beta_0$; thus, $\vec{b}$ can be written as $\vec{b} = \frac{4}{9\beta_0}$.

[13] The solution of the condition $b = \vec{q}_e$ in terms of b yields the following two roots:

$b_1 = \frac{1+9\alpha\Pi_m + \sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2}}{2\alpha}$ and $b_2 = \frac{1+9\alpha\Pi_m - \sqrt{1+36\alpha C_p^T - 18\alpha\Pi_m + 81\alpha^2\Pi_m^2}}{2\alpha}$.

The root b_1 is rejected as a possible solution since $b_1 > 1 \ \forall \Pi_m \in (C_p^T, \frac{1}{9} + \frac{C_p^T}{1-\alpha}), \Pi_m > 0, C_p^T \geq 0, \alpha \in (0,1)$. The

- The Effect of α, C_p^T and Π_m on β_0.

$$\frac{\partial \beta_0}{\partial \alpha}=\frac{-8\alpha(9\Pi_m-\dfrac{162\alpha\Pi_m^2-18\Pi_m+36C_p^T}{2\sqrt{1+36\alpha C_p^T-18\alpha\Pi_m+81\alpha^2\Pi_m^2}})+8(1+9\alpha\Pi_m-\sqrt{1+36\alpha C_p^T-18\alpha\Pi_m+81\alpha^2\Pi_m^2})}{9(1+9\alpha\Pi_m-\sqrt{1+36\alpha C_p^T-18\alpha\Pi_m+81\alpha^2\Pi_m^2})^2}\geq 0$$

$\forall \alpha\in(0,1), C_p^T\geq 0 \ \wedge \ \Pi_m\in(C_p^T, \frac{1}{9}+\frac{C_p^T}{1-\alpha})$.

$$\frac{\partial \beta_0}{\partial C_p^T}=\frac{16\alpha^2}{(\sqrt{1+36\alpha C_p^T-18\alpha\Pi_m+81\alpha^2\Pi_m^2})(-1-9\alpha\Pi_m+\sqrt{1+36\alpha C_p^T-18\alpha\Pi_m+81\alpha^2\Pi_m^2})^2}\geq 0$$

$\forall \alpha\in(0,1), C_p^T\geq 0 \ \wedge \ \Pi_m\in(C_p^T, \frac{1}{9}+\frac{C_p^T}{1-\alpha})$.

$$\frac{\partial \beta_0}{\partial \Pi_m}=\frac{8\alpha(\dfrac{162\alpha^2\Pi_m-18\alpha}{2\sqrt{1+36\alpha C_p^T-18\alpha\Pi_m+81\alpha^2\Pi_m^2}}-9\alpha)}{9(\sqrt{1+36\alpha C_p^T-18\alpha\Pi_m+81\alpha^2\Pi_m^2}-1-9\alpha\Pi_m)^2}\leq 0 \ \ \forall \alpha\in(0,1), C_p^T\geq 0 \ \wedge \ \Pi_m\in(C_p^T, \frac{1}{9}+\frac{C_p^T}{1-\alpha}).$$

$$\frac{\partial^2 \beta_0}{\partial \Pi_m{}^2}=-\frac{16\alpha(\dfrac{162\alpha^2\Pi_m-18\alpha}{2\sqrt{1+36\alpha C_p^T-18\alpha\Pi_m+81\alpha^2\Pi_m^2}}-9\alpha)^2}{9(\sqrt{1+36\alpha C_p^T-18\alpha\Pi_m+81\alpha^2\Pi_m^2}-1-9\alpha\Pi_m-)^3}+$$

$$\frac{8\alpha(\dfrac{(162\alpha^2\Pi_m-18\alpha)^2}{4(1+36\alpha C_p^T-18\alpha\Pi_m+81\alpha^2\Pi_m^2)^{3/2}}+\dfrac{18\alpha^2}{\sqrt{1+36\alpha C_p^T-18\alpha\Pi_m+81\alpha^2\Pi_m^2}})}{9(\sqrt{1+36\alpha C_p^T-18\alpha\Pi_m+81\alpha^2\Pi_m^2}-1-9\alpha\Pi_m)^2}\geq 0$$

$\forall \alpha\in(0,1), C_p^T\geq 0 \ \wedge \ \Pi_m\in(C_p^T, \frac{1}{9}+\frac{C_p^T}{1-\alpha})$.

The above imply that, the greater are the validity parameter, α, and the incumbent's trial costs, C_p^T, and the smaller are the monopoly profits, Π_m, the greater is the critical value β_0 and the more likely it is that patenting will not be an optimal strategy for the incumbent.

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root b_2 is accepted as a possible solution as $b_2\in(0,1)$ for $\Pi_m\in(C_p^T, \frac{1}{9}+\frac{C_p^T}{1-\alpha}), \Pi_m>0, C_p^T\geq 0, \alpha\in(0,1)$. Given the above $b_2=\vec{b}$.

B_2. Conditions under which a $\hat{b}$ that deters entry exists

From proposition 2 we know that entry deterrence requires that $(\Pi_e^I)^{NT}(\vec{b}) \leq 0 \wedge E(\Pi_e^I)^T(\tilde{b}) < 0$. It is straightforward to show that $(\Pi_e^I)^{NT}(\vec{b}) = 0$ when $\beta = 2\beta_0$ where β_0 is defined in Appendix B_1. Since $(\Pi_e^I)^{NT} = \frac{4}{9}\vec{q}_e - \frac{\beta}{2}\vec{q}_e^{\,2}$ is decreasing in β, $(\Pi_e^I)^{NT}(\vec{b}) \leq 0$ for $\beta \geq 2\beta_0$. Thus, the condition $\beta \geq 2\beta_0$ is one of the necessary conditions for entry deterrence. The other condition requires that at $\tilde{b}$ $E(\Pi_e^I)^T(\tilde{b}) \leq 0$.

Substitution of $\tilde{b}$ into the entrant's profit function under entry, infringement and trial, yields

$E(\Pi_e^I)^T(\tilde{b}) = \frac{512\alpha^2}{6561\beta^3} - C_e^T$ and thus, $E(\Pi_e^I)^T(\tilde{b}) = \frac{512\alpha^2}{6561\beta^3} - C_e^T \leq 0$ when $C_e^T \geq \frac{512\alpha^2}{6561\beta^3} = \hat{C}_e^T$.[14] Given

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[14] If a patent breadth $\tilde{b}$ that makes the entrant indifferent between infringing and not infringing the patent exists, it should satisfy the condition $\Pi_e^{NI}(\tilde{b}) = E(\Pi_e^I)^T(\tilde{b}))$ and it should take values $\tilde{b} \in (b_0, \bar{b}]$ when $\bar{b} \in (\vec{b}, 1]$ exists and $\tilde{b} \in (b_0, 1]$ when $\bar{b} \in (\vec{b}, 1]$ does not exist. The solution of $\Pi_e^{NI}(\tilde{b}) = E(\Pi_e^I)^T(\tilde{b}) \Rightarrow (\frac{8\alpha^2}{81\beta} + \frac{\beta}{2})\tilde{b}^2 - \frac{4}{9}\tilde{b} - C_e = 0$ in terms of $\tilde{b}$ yields the following two roots: $\tilde{b}_{1,2} = \frac{9(4\beta \pm \sqrt{2}\sqrt{\beta}\sqrt{16C_e^T\alpha^2 + 8\beta + 81C_e^T\beta^2})}{16\alpha^2 + 81\beta^2}$.The root $\tilde{b}_1 = \frac{9(4\beta - \sqrt{2}\sqrt{\beta}\sqrt{16C_e^T\alpha^2 + 8\beta + 81C_e^T\beta^2})}{16\alpha^2 + 81\beta^2} \leq 0 \;\; \forall \;\; \beta > \frac{4}{9},\; \alpha \in (0,1) \wedge C_e^T \geq 0$ and it is thus rejected. The root $\tilde{b}_1 = \frac{9(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_e^T\alpha^2 + 8\beta + 81C_e^T\beta^2})}{16\alpha^2 + 81\beta^2} \geq 0$ for $\beta > \frac{4}{9},\; \alpha \in (0,1) \wedge C_e^T \geq 0$ and it is accepted as a possible solution. If $\tilde{b} = \frac{9(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_e^T\alpha^2 + 8\beta + 81C_e^T\beta^2})}{16\alpha^2 + 81\beta^2}$ exists it should also satisfy the condition $b_0 < \tilde{b} \leq 1$ if $\bar{b} \in (\vec{b}, 1]$ does not exist or the condition $b_0 < \tilde{b} < \bar{b}$ if $\bar{b} \in (\vec{b}, 1]$ exists. The condition $\tilde{b} > b_0$ is satisfied since $\tilde{b} - b_0 = \frac{9(4\beta + \sqrt{2}\sqrt{\beta}\sqrt{16C_e^T\alpha^2 + 8\beta + 81C_e^T\beta^2})}{16\alpha^2 + 81\beta^2} - \frac{4}{9\beta} > 0 \;\; \forall \;\; \beta > \frac{4}{9},\; \alpha \in (0,1) \wedge C_e^T \geq 0$. When $\bar{b} \in (\vec{b}, 1]$ does not exist, the condition $\tilde{b} \leq 1$ is satisfied for certain combinations of β, α and C_e^T values. To determine the combinations of β, α and C_e^T values which satisfy the condition $\tilde{b} \leq 1$, the pairs of β, α and C_e^T values that satisfy the above constraint as an equality ($\tilde{b} = 1$) are determined first. The solution of $\tilde{b} = 1$ with respect to C_e^T yields $C_e^T = \frac{16\alpha^2 - 72\beta + 81\beta^2}{162\beta}$. The area to the right of the locus $\tilde{b} = 1$ represents all combinations of β and C_e^T values, for a given α value, for which $\tilde{b} < 1$. When $\bar{b} \in (\vec{b}, 1]$ exists, the condition $\tilde{b} < \bar{b}$ is satisfied when $C_e^T < \frac{(4 + 81\beta C_p^T - 81\beta\Pi_m)(-288\alpha\beta + (16\alpha^2 + 81\beta^2)(81\beta C_p^T + 4) - 81\beta(4\alpha - 9\beta)^2\Pi_m)}{162\alpha^2\beta(4 - 81\beta\Pi_m)^2}$.

the above, the conditions $\beta \geq 2\beta_0$ and $C_e^T \geq \dfrac{512\alpha^2}{6561\beta^3} = \hat{C}_e^T$ are the necessary and sufficient conditions for entry deterrence.

B_3. Conditions under which entry cannot be deterred

- Case I_A: $(\Pi_e^I)^{NT}(\vec{b}) \geq E(\Pi_e^I)^T(\vec{b})$ and $\bar{b} \in (\vec{b}, 1]$ exists

Under this case $(\Pi_e^I)^{NT}(\vec{b}) > 0$, $(\Pi_e^I)^{NT}(\vec{b}) \geq E(\Pi_e^I)^T(\vec{b})$ and $(\Pi_e^I)^{NT}(b=1) > E(\Pi_e^I)^T(b=1)$, which imply that this case for $\beta < 2\beta_0$, $C_e^T \geq (\dfrac{8\alpha^2}{81\beta} + \dfrac{\beta}{2})\vec{b}^2 - \dfrac{4}{9}\vec{b} \Rightarrow C_e^T \geq (\dfrac{8\alpha^2}{81\beta} + \dfrac{\beta}{2})\dfrac{16}{81\beta_0^{\ 2}} - \dfrac{16}{81\beta_0} = \tilde{C}_e^T$ and

$$C_e^T > \frac{8\alpha^2}{81\beta} + \frac{81\beta}{2}(\Pi_m - \frac{C_p^T}{1-\alpha})^2 - 4(\Pi_m - \frac{C_p^T}{1-\alpha}) = \tilde{\tilde{C}}_e^T .$$

- Case I_B: $(\Pi_e^I)^{NT}(\vec{b}) \geq E(\Pi_e^I)^T(\vec{b})$ and $\bar{b} \in (\vec{b}, 1]$ does not exist

Under this case $(\Pi_e^I)^{NT}(\vec{b}) > 0$, $(\Pi_e^I)^{NT}(\vec{b}) \geq E(\Pi_e^I)^T(\vec{b})$ and $(\Pi_e^I)^{NT}(b=1) < E(\Pi_e^I)^T(b=1)$. These conditions imply that $\beta < 2\beta_0$, $C_e^T \geq \tilde{C}_e^T$ and $C_e^T < \tilde{\tilde{C}}_e^T$.

- Case II_A: $(\Pi_e^I)^{NT}(\vec{b}) < E(\Pi_e^I)^T(\vec{b})$ and $\bar{b} \in (\vec{b}, 1]$ exists

This case can emerge both when at $\vec{b}$ $(\Pi_e^I)^{NT}(\vec{b}) > 0$ which implies that $\beta < 2\beta_0$ and when at $\vec{b}$ $(\Pi_e^I)^{NT}(\vec{b}) \leq 0$ which implies that $\beta \geq 2\beta_0$. Under this case, at $\tilde{b}$ $\Pi_e^{NI}(\tilde{b}) = E(\Pi_e^I)^T(\tilde{b})) > 0$ which implies that $C_e^T < \hat{C}_e^T$. Also, at $\vec{b}$ $(\Pi_e^I)^{NT}(\vec{b}) < E(\Pi_e^I)^T(\vec{b})$; in addition $(\Pi_e^I)^{NT}(b=1) > E(\Pi_e^I)^T(b=1)$ which together imply that $C_e^T < \tilde{C}_e^T$ and $C_e^T > \tilde{\tilde{C}}_e^T$, respectively.

- Case II_B: $(\Pi_e^I)^{NT}(\vec{b}) < E(\Pi_e^I)^T(\vec{b})$ and $\bar{b} \in (\vec{b}, 1]$ does not exist

This case can emerge both when at $\vec{b}$ $(\Pi_e^I)^{NT}(\vec{b}) > 0$ which implies that $\beta < 2\beta_0$ and when $(\Pi_e^I)^{NT}(\vec{b}) \leq 0$ which implies that $\beta \geq 2\beta_0$. Under this case, at $\tilde{b}$ $\Pi_e^{NI}(\tilde{b}) = E(\Pi_e^I)^T(\tilde{b})) > 0$ which implies that $C_e^T < \hat{C}_e^T$. Also, at $\vec{b}$ $(\Pi_e^I)^{NT}(\vec{b}) < E(\Pi_e^I)^T(\vec{b})$ and $(\Pi_e^I)^{NT}(b=1) < E(\Pi_e^I)^T(b=1)$ (i.e., a $\bar{b} \in (\vec{b}, 1]$ does not exist) which imply that $C_e^T < \tilde{C}_e^T$ and $C_e^T < \tilde{\tilde{C}}_e^T$, respectively.

Note that when $\beta = 2\beta_0$, the loci $\hat{C}_e^T = \tilde{C}_e^T = \frac{64a^2}{6561\beta_0^3} = \bar{C}_e^T$ where $\bar{C}_e^T = \frac{64a^2}{6561\beta_0^3}$ is the value of the entrant's trial costs that makes his expected profits under infringement and trial equal to zero at $\vec{b}$. Also, the locus $\tilde{\tilde{C}}_e^T$ may be greater, smaller or equal to the locus $\tilde{C}_e^T$ at $\beta = 2\beta_0$ depending on the value of the exogenous parameters. Figure A.2 depicts the locus $\tilde{\tilde{C}}_e^T$ crossing the locus $\tilde{C}_e^T$ so that cases $\mathrm{I_B}$ and $\mathrm{II_A}$ are both feasible.