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Decomposition of structural and exogenous changes

GEORGE W. LADD and STEVEN S. DUNCAN

Department of Economics, Iowa State University, Ames, Iowa 50011, USA

Tests and measures of structural change are common in econometric work. Our profession is well equipped with tests for structural change (see, e.g., Brown, Durbin and Evans, 1975; Chow, 1965; Dhrymes *et al.*, 1972). Many writers have emphasized the importance of the ability to predict structural change. In the early post-World War II years, writers associated with and influenced by the Cowles Commission were promoting simultaneous equations estimation as being necessary for prediction under changed structure. As long as structure remained unchanged, ordinary least squares estimates of reduced form coefficients were adequate; but when a change in structure occurred it was necessary to estimate the structural coefficients in order to incorporate the structural change. Examples of this argument are found in Girshick and Haavelmo (1947), Koopmans (1949), and Marschak (1947). More recently the use of rational expectations hypothesis has been justified on similar grounds: it provides a way of measuring the structural changes that are brought about by changes in public policy. This is essentially the Lucas critique (Lucas, 1976, see Fisher, 1982 for an example). Unfortunately our recognition of the need to predict structural changes greatly exceeds our ability to predict them. Prediction of structural change seems more difficult at the micro level than at the macro level. For example, we are probably less successful at prediction of the effect of a change in social security tax withholdings upon beef demand equations than upon total consumer expenditures.

Usually tests of structural changes are not an end in themselves but are only a means to an end of understanding changes in values of endogenous variables. To achieve that end it is not sufficient to look only at changes in coefficients. If $y=bx$, then $\mathrm{d}y=b\mathrm{d}x+x\mathrm{d}b+\mathrm{d}b\mathrm{d}x$. The tests only provide information on $\mathrm{d}b$. But the effect of $\mathrm{d}b$ on $\mathrm{d}y$ depends on the values of x and of $\mathrm{d}x$. A highly significant change in b may generate a small change in y.

The effect of $\mathrm{d}b$ is quite different if x is in the expansion phase of its cycle than if it is in the contraction phase or if it is at a turning point. When the main concern is with understanding causes of $\mathrm{d}y$, and testing of $\mathrm{d}b$ is only a means to this end, we suggest decomposing $\mathrm{d}y$ into its components.

Also, a change in the value of a coefficient can increase or decrease the variance of an endogenous variable. If $y=b_1x_1+b_2x_2$, the change in the variance of y induced by a change in the value of b_1 is

$$\mathrm{d}V(y)=V(x_1)\mathrm{d}b_1^2+2b_2\,\mathrm{Cov}(x_1,\,x_2)\mathrm{d}b_1$$

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where db_1^2 is the change in the square of b_1. It can also be informative, therefore, to decompose the change in the variance of an endogenous variable.

This paper outlines the decomposition of a mean change, the calculation of the variance of a mean change, and the decomposition of a variance change, all for the static model. A decomposition for a dynamic model is also discussed. Finally, a numerical example is presented to illustrate the techniques. Our procedures can be used whenever statistical tests indicate that structural change has occurred; that is, they can be used to study the past. They can also be used when future structural change is predicted.

The term 'structural change' is relative: relative to the structure specified. If we have specified a linear relation between y and x we might have to conclude that the relation is different in recent years than it once was, possibly $y = a + bx$ for early years but $y = a' + b'x$ recently. If, however, recent values of x are larger than earlier values, we might cover all observations by specifying $y = \alpha + \beta x + \gamma x^2$. In this example, the linear relation undergoes structural change; the quadratic does not.

One can imagine models that eliminate virtually all structural change and specification error. Rausser, Mundlak and Johnson (1982) present relations such as

$$y_t = \sum_i b_{it} x_{it} + e_t$$

in which the coefficients undergo systematic and random variation,

$$b_{it} = b_{i0} + \sum_j a_{ij} x_{jt} + \sum_h c_{ih} Z_{ht} + r_t$$

where the Z_{ht} can represent lags and dichotomies. Mellor and Hessner (1986) used a similar, but somewhat simpler, formulation in a Markov chain model of structural changes in the demand for butter and margarine. Such procedures virtually eliminate chances of 'specification error of omission', but increase chances of 'specification error of inclusion'. They increase the chance of type II error: type II error would become quite likely in data mining studies that estimated a number of combinations of functional forms and explanatory variables in order to explain structural change.

The issue that we are dealing with in this paper – the allocation of the changes in an endogenous variable among the various sources of changes – arises whenever one chooses to model structural change.

I. STATIC MODEL

First we consider linear functions. The equation is assumed to have but one endogenous variable. If the equation is one in a simultaneous system, the coefficients are derived reduced-form coefficients. Decomposition of a single equation system will be discussed before decomposition of one of a system of simultaneous equations.

Change in mean

For concreteness, suppose we are comparing the mean value of y after a structural change with the mean value before a structural change. Let y_{st} be the value of the endogenous variable in time period t in sample period s; $s = 1$ for the period before the structural change

and $s=2$ after the change. Assume that the equation's disturbances are temporally independent and homoscedastic in each sample period. Let x_{ist} be the value of the ith exogenous variable in period s, time period t. The estimated equation for sample period s is $y_{st}=\sum_{i=1}^{k} b_{is}x_{ist}+e_{st}$. Let $\bar{y}_1$ and $\bar{x}_{i1}$ be the mean values of the endogenous variable and the ith exogenous variable in the period before the change and $\bar{y}_2$ and $\bar{x}_{i2}$ be the means after the change, and let b_{i1} and b_{i2} be the coefficients of the ith exogenous vairable before and after the change. Define

$$\mathrm{d}\bar{y}=\bar{y}_2-\bar{y}_1,$$
$$\mathrm{d}\bar{x}_i=\bar{x}_{i2}-\bar{x}_{i1},$$
$$\mathrm{d}b_i=b_{i2}-b_{i1}.$$

Then we have exactly

$$\mathrm{d}\bar{y}\equiv\sum_i \bar{x}_{i1}\,\mathrm{d}b_i+\sum_i b_{i1}\,\mathrm{d}\bar{x}_i+\sum_i \mathrm{d}\bar{x}_i\mathrm{d}b_i \tag{1}$$

The first term equals the change in mean y due to changes in coefficients with exogenous variables constant at their period one means. The second equals the change due to changes in exogenous variables if coefficients are held constant at their period one values. The third equals the change in mean y due to interactions between changes in coefficients and changes in variables. The change in y can be further decomposed to isolate effects of particular changes. For example, setting $\mathrm{d}\bar{x}_i=\mathrm{d}b_i=0$ for all $i\geqslant 2$ isolates the effects of changes in b_1 and $\bar{x}_1$.

It is also possible and desirable to measure the reliability of our explanation of $\mathrm{d}\bar{y}$. To obtain a standard error of forecast of $\mathrm{d}\bar{y}$, define

$$\bar{X}'_1=(\bar{x}_{11},\bar{x}_{21},\ldots,\bar{x}_{k1})$$
$$\mathrm{d}\bar{X}'=(\mathrm{d}\bar{x}_1,\mathrm{d}\bar{x}_2,\ldots,\mathrm{d}\bar{x}_k)$$
$$\mathrm{d}b=(\mathrm{d}b_1,\mathrm{d}b_2,\ldots,\mathrm{d}b_k)'$$
$$b=(b_{11},b_{21},\ldots,b_{k1})'$$

Then

$$\mathrm{d}\bar{y}=(\bar{X}'_1,\mathrm{d}\bar{X}',\mathrm{d}\bar{X}')\begin{pmatrix}\mathrm{d}b\\ b\\ \mathrm{d}b\end{pmatrix}$$

Computations can be simplified by noting that

$$(\bar{X}'_1,\mathrm{d}\bar{X}',\mathrm{d}\bar{X}')=(\bar{X}'_1,\mathrm{d}\bar{X}')\left(\begin{array}{c:c} I_{2k} & 0_k\\ \hdashline & I_k\end{array}\right)$$

where I_k and I_{2k} are k by k and $2k$ by $2k$ identity matrices, and 0_k is a k by k null matrix.

$$\begin{pmatrix}\mathrm{d}b\\ b\\ \mathrm{d}b\end{pmatrix}=\left(\begin{array}{c:c} I_{2k} & \\ \hdashline I_k & 0_k\end{array}\right)\begin{pmatrix}\mathrm{d}b\\ b\end{pmatrix}$$

Now we can write

$$\begin{aligned} d\bar{y} &= (\bar{X}'_1, d\bar{X}') \begin{pmatrix} I_k & 0_k \\ I_k & I_k \end{pmatrix} \begin{pmatrix} db \\ b \end{pmatrix} \\ &= (\bar{X}'_1, d\bar{X}') J \begin{pmatrix} db \\ b \end{pmatrix} = \bar{Z}' B \end{aligned} \tag{2}$$

say, where $\bar{Z}' = (\bar{X}'_1\, d\bar{X}')J$. If we are studying past changes, $\bar{Z}'$ is a known vector and the variance of $d\bar{y}$ is

$$V(d\bar{y}) = \bar{Z}' D(B) \bar{Z} \tag{3}$$

where $D(B)$ is the symmetric matrix of variances and covariances of elements of B.

$$D(B) = \begin{bmatrix} V(db_1) & C(db_1, db_2) & \dots & C(db_1, db_k) & C(db_1, b_{11}) & \dots & C(db_1, b_{k1}) \\ & V(db_2) & \dots & C(db_2, db_k) & C(db_2, b_{11}) & \dots & C(db_2, b_{k1}) \\ & & & \vdots & & & \\ & & & V(db_k) & C(db_k, b_{11}) & \dots & C(db_k, b_{k1}) \\ & & & & V(b_{11}) & \dots & C(b_{11}, b_{k1}) \\ \text{symm.} & & & & & & \vdots \\ & & & & & & V(b_{k1}) \end{bmatrix}$$

The terms $V(b_i)$ and $C(b_i, b_j)$ are available from the original estimation. Computation of the other elements is straightforward if errors in the two periods are independent. Then

$$\begin{aligned} V(db_i) &= V(b_{i1}) + V(b_{i2}) \\ C(db_i, db_j) &= C(b_{i1}, b_{j1}) + C(b_{i2}, b_{j2}) \\ C(db_i, b_{j1}) &= -C(b_{i1}, b_{j1}) \\ C(db_i, b_{i1}) &= -V(b_{i1}). \end{aligned}$$

A likely situation is one in which period one is the recent past and period two is the near future. Then the previous formula for $V(dy)$ is not relevant because it treats $d\bar{X}$ as known, but $d\bar{X}$ must be forecast. The proper formula for variance of a forecast must now contain $D(\bar{Z})$. $\bar{X}_1$ is a known constant vector. So

$$D(\bar{X}'_1, d\bar{X}') = \begin{pmatrix} 0_k & 0_k \\ 0_k & D(d\bar{X}) \end{pmatrix}$$

Making use of the assumption of independent errors and the fact that $\bar{X}_1$ is a constant vector yields as the elements of $D(d\bar{X})$

$$\begin{aligned} V(d\bar{x}_i) &= V(\bar{x}_{i2}) \\ C(d\bar{x}_i, d\bar{x}_j) &= C(\bar{x}_{i2}, \bar{x}_{j2}) \\ C(d\bar{x}_i, \bar{x}_{j1}) &= 0 \\ C(d\bar{x}_i, \bar{x}_{i1}) &= 0 \end{aligned}$$

where $V(\bar{x}_{i2})$ and $C(\bar{x}_{i2}, \bar{x}_{j2})$ are variance and covariance of forecasts of means. Then

$$D(\bar{Z}) = J'_{2k} D(\bar{X}_1, d\bar{X}) J_{2k}$$

And

$$V(\mathrm{d}\bar{y}) = \bar{Z}'D(B)\bar{Z} + B'D(\bar{Z})B + \mathrm{tr}\,[D(B)D(\bar{Z})] \tag{4}$$

Note that the error variances for the two periods σ_2^2 and σ_1^2 do not enter this equation because we are dealing with variance of forecasts of means.

Other decompositions can be done in a similar fashion. One might, for example, compare each calendar quarter in the last full year of sample period one with the corresponding quarter in the first full year in sample period 2. Using the subscript q ($=1, 2, 3, 4$) to identify calendar quarters, each decomposition might be written

$$\mathrm{d}\bar{y}_q = \sum_i \bar{x}_{i1q}\mathrm{d}b_i + \sum_i b_{i1}\mathrm{d}\bar{x}_{iq} + \sum_i \mathrm{d}b_i\mathrm{d}\bar{x}_{iq}.$$

The preceding formulas hold whether the equation is a single equation or part of a simultaneous system. But the latter situation needs a bit more discussion. If, for example, the only changes in a single-equation model are the coefficient of x_1 and the mean of x_1, we need only compute

$$\mathrm{d}\bar{y} = \bar{x}_{11}\mathrm{d}b_1 + b_{11}\mathrm{d}\bar{x}_1 + \mathrm{d}\bar{x}_1\mathrm{d}b_1$$

If one structural coefficient in a system changes, the situation is different because the value of each derived reduced form coefficient depends on values of several (or all) structural coefficients.

Write the system of simultaneous structural equations for sample period s as

$$A_s Y_s(t) = C_s X_s(t) + U_s(t),$$

and

$$Y_s(t) = A_s^{-1} C_s X_s(t) + A_s^{-1} U_s(t).$$

The elements of $A_s^{-1}C_s$ are the b_{is}. Suppose we only want to investigate the effect of a change in one structural coefficient, say a_{uv}: the coefficient of $y_v(t)$ in the uth structural equation. Write $B_s = A_s^{-1}C_s$ and $\mathrm{d}B = B_2 - B_1$. Also, we can write $A_2 = A_1 + J\mathrm{d}a_{uv}$ where J has unity in row u and column v and zeros elsewhere. Then

$$\mathrm{d}B = A_2^{-1}C_2 - A_1^{-1}C_1 = (A_1 + J\mathrm{d}a_{uv})^{-1}C_2 - A_1^{-1}C_1$$

gives values of all the $\mathrm{d}b_i$. Now not only does a change in one structural coefficient affect several or all reduced-form coefficients, it has an effect on several or all endogenous variables. This argument easily extends to changes in several structural coefficients by setting $A_2 = A_1 + \mathrm{d}A$ and $C_2 = C_1 + \mathrm{d}C$. Then

$$\mathrm{d}b = (A_1 + \mathrm{d}A)^{-1}(C_1 + \mathrm{d}C) - A_1^{-1}C_1.$$

Equations 1 and 2 can be used to decompose the mean change in each endogenous variable. Equations 3 and 4 provide variances of the decompositions.

We now extend the argument to cover situations of known non-linearities in exogenous variables: where y can be written as a sum of known functions of exogenous variables, $y = \sum_i b_i f(x_i)$. For example, $f(x_1) = \ln x_1$, $f(x_2) = x_2^2$, $f(x_{1,2}) = (\ln x_1)x_2^2$; but not $f(x_1) = x_1^a$ where a is estimated and changes value between sample periods one and two. Denote $\bar{f}(x_{is})$ as the mean value of $f(x_{is})$ in period s, and $\mathrm{d}\bar{f}(x_i) = \bar{f}(x_{i2}) - \bar{f}(x_{i1})$. If $f(x_{is}) = \ln x_{is}$, then

$\bar{f}(x_{is})=$ mean $(\ln x_{is})$ and $\mathrm{d}\bar{f}(x_i)=$ mean $(\ln x_{i2})-$ mean $(\ln x_{i1})$. If $f(x_{is})=x_{is}^2$, then $\bar{f}(x_{is})$ $=$ mean (x_{is}^2). Equation 1 is replaced by

$$\mathrm{d}\bar{y}=\sum_i \bar{f}(x_{i1})\mathrm{d}b_i+\sum_i b_{i1}\,\mathrm{d}\bar{f}\,(x_i)+\sum_i \mathrm{d}b_i\mathrm{d}\bar{f}(x_i). \tag{5}$$

Redefine $\bar{X}_1$ as $(\bar{f}(x_{11}),\bar{f}(x_{21}),\ldots,\bar{f}(x_{k1}))$ and redefine $\mathrm{d}\bar{X}$ and $\bar{Z}$ appropriately. Then Equation 2 provides $\mathrm{d}\bar{y}$. If sample periods one and two are past, then the x_{i1} and x_{i2} are known. Because the $f(x_{is})$ are known functions, $\bar{f}(x_1)$ and $\bar{f}(x_2)$ are known, and the variance of $\mathrm{d}\bar{y}$ is provided by Equation 3.

If period two is a future period, the x_{i2} must be predicted, and the values of $f(x_{i2})$ and $\bar{f}(x_{i2})$ computed. It will not do to predict $\bar{x}_{i2}$ and then compute $f(\bar{x}_{i2})$ because $\bar{f}(x_{i2})\neq f(\bar{x}_{i2})$. Formulas for variances and covariances of products and/or statistical differentials can be used to estimate $V(\bar{f}(x_{i2}))$ and $C(\bar{f}(x_{i2}),\bar{f}(x_{j2}))$, and the properly redefined $D(\bar{Z})$ can be obtained. Then Equation 4 can be applied.

Change in variance

We can also decompose the change in the variance. $V(y_s)=\sum_i\sum_j b_{is}b_{js}\,C(x_{is},x_{js})+V(e_s)$. Letting $V(y_s)$ and $C(x_{is},x_{js})$ be variance of y and covariance between exogenous variables in period s, and $V(e_s)$ be residual variance, we have

$$\begin{aligned}\mathrm{d}V(y)=&\sum_i\sum_j C(x_{i1},x_{j1})\,d(b_ib_j)\\&+\sum_i\sum_j b_{i1}b_{j1}\,\mathrm{d}C(x_i,x_j)\\&+\sum_i\sum_j d(b_ib_j)\,\mathrm{d}C(x_i,x_j)+\mathrm{d}V(e)\end{aligned} \tag{6}$$

where $\mathrm{d}V(y)=V(y_2)-V(y_1)$, $d(b_ib_j)=b_{i2}b_{j2}-b_{i1}b_{j1}$, $\mathrm{d}C(x_i,x_j)$ $=C(x_{i2},x_{j2})-C(x_{i1},x_{j1})$.

If the only change is in the value of b_{11},

$$\mathrm{d}V(y)=(b_{12}^2-b_{11}^2)\,V(x_{11})$$

If the only change is in the variance of x_1,

$$\mathrm{d}V(y)=b_{11}^2\,(V(x_{12})-V(x_{11}))$$

If the variance of x_1 changes, it is also likely that its covariances change. If the only changes that occur are in the variance and covariances of x_1, we have

$$\mathrm{d}V(y)=b_{11}^2\,\mathrm{d}V(x_1)+2\sum_{j\neq 1} b_{11}b_{j1}\,\mathrm{d}C(x_1,x_j)$$

II. DYNAMIC MODEL

Decomposing a dynamic model is not so straightforward. The existence of lags means that the effects of a change in an exogenous variable are spread out over time. One can generate

and decompose two synthetic time series and can also compare multipliers. Suppose the system for the tth observation in sample period s is

$$\mathbf{Y}_{st} = A_s \mathbf{Y}_{st-1} + B_s \mathbf{X}_{st} + \mathbf{U}_{st}$$

where $\mathbf{Y}_{st}$, $\mathbf{Y}_{st-1}$, $\mathbf{X}_{st}$, and $\mathbf{U}_{st}$ are vectors and A and B are matrices. Let $\bar{\mathbf{X}}_s$ denote the vector of mean values of the exogenous variables for sample period s, and $\mathbf{Y}_{s0}$ represent the vector of initial conditions for sample period s. Assume the system is stable. Its solution is

$$\mathbf{Y}_{st} = A_s^t \mathbf{Y}_{s0} + \sum_{i=0}^{t-1} A_s^i B_s \mathbf{X}_{st-i} + \sum_{i=0}^{t-1} A_s^i \mathbf{U}_{st-i}$$

Letting all $\mathbf{U}_{st-i}$ equal to zero, we can generate two vectors of synthetic time series

$$\bar{Y}_{st} = A_s^t \mathbf{Y}_{s0} + \sum_{i=0}^{t-1} A_s^i B_s \bar{\mathbf{X}}_s$$

The vectors can be decomposed as

$$\begin{aligned} \bar{\mathbf{Y}}_{2t} - \bar{\mathbf{Y}}_{1t} = {} & (A_2^t - A_1^t)\, Y_{10} + \sum_i (A_2^i B_2 - A_1^i B_1) \bar{X}_1 \\ & + A_1^t (Y_{20} - Y_{10}) + \sum_i A_1^i B_1 (\bar{X}_2 - \bar{X}_1) \\ & + (A_2^t - A_1^t)(Y_{20} - Y_{10}) + \sum_i (A_2^i B_2 - A_1^i B_1)(\bar{X}_2 - \bar{X}_1) \end{aligned} \tag{7}$$

The first two terms on the right-hand side of this expression represent differences in $\bar{\mathbf{Y}}_{2t}$ and $\bar{\mathbf{Y}}_{1t}$ caused by changes in coefficients, with fixed initial conditions and fixed exogenous variables. The second two terms are the differences due to changes in initial conditions and exogenous variables, with constant coefficients. The last two terms equal differences due to interactions between changes in coefficients, and changes in initial conditions and exogenous variables.

The effects of structural change upon multipliers can also be obtained. B_s is the matrix of impact multipliers for sample period s. $A_s^d B_s$ is the matrix of d-period delay multipliers for period s; $\partial y_{it}/\partial x_{jt-d}$. $\sum_{i=0}^{d} A_s^i B_s$ is the matrix of d-period cumulative multipliers: $\sum_{i=0}^{d} \partial y_{it}/\partial x_{jt-i}$. The equilibrium multiplier matrix for a stable system is $\sum_{i=0}^{\infty} A_s^i B_s = (I - A_s)^{-1} B_s$.

III. NUMERICAL EXAMPLE

The annual average US farm-retail beef and pork marketing margins increased by 45 and 30%, respectively, between 1974–77 and 1978–81. They reached record highs in 1976–77, and then increased by 12% between 1977 and 1978. Their variances also increased. In an effort to explain these changes, we used monthly data to estimate a 12-equation model of the wholesale–retail and farm–wholesale beef and pork marketing sector. We also estimated smaller models.

A farm–retail marketing margin is the difference between a composite retail price for the various cuts of meat and the net farm value of an equivalent amount of live animal. The

farm–retail marketing margin can be broken into the farm–wholesale and the wholesale–retail marketing margins. The margin can be viewed as the cost of performing all of the activities involved in transportation, slaughtering, processing, wholesaling, and retailing.

Structural change can be gradual or abrupt. Because margins changed so dramatically between 1977 and 1978, we assumed that abrupt structural change occurred and was completed between 1977 and 1978, and we used two sample periods. The first was 1968–77 and the second was 1978–84. An alternative would have been to use a model like Sheflin's (1984) procedure for estimation of a transition function. One use of our model was to identify coefficient structural change in the demand and margin equations. The block-recursive model consisted of two real-price dependent demand equations, two identities to link nominal and real prices, four margin equations, and four identities to link retail and farm prices. Since the system of equations was block-recursive, a modification of the preceding discussion was needed to determine all effects of changes in coefficients in the first block. For the numerical example, we wrote the block-recursive system for sample period s as

$$\begin{bmatrix} I & 0 & 0 \\ -I & D_{2s} & 0 \\ 0 & E_{2s} & E_{3s} \end{bmatrix} \begin{bmatrix} \mathbf{Y}_{1s} \\ \mathbf{Y}_{2s} \\ \mathbf{Y}_{3s} \end{bmatrix} = \begin{bmatrix} F_{1s} \\ 0 \\ F_{3s} \end{bmatrix} X_s + \begin{bmatrix} u_{1s} \\ 0 \\ u_{3s} \end{bmatrix} \tag{8}$$

$\mathbf{Y}_{1s}$ is a (2×1) vector of the real retail prices of beef and pork. $\mathbf{Y}_{2s}$ is a (2×1) vector of the nominal retail prices of beef and pork (PB and PP). $\mathbf{Y}_{3s}$ is an (8×1) vector of the wholesale–retail and the farm–wholesale margins and the farm and wholesale values of beef and pork. Line one of Equation 8 is the two demand equations. D_{2s} is the reciprocal of the Consumer Price Index (CPI) multiplied by a (2×2) identity matrix and so line two of Equation 8 is the two identities that link the real and nominal prices. Line three contains the four margin equations and the four identities that link farm and wholesale values and retail prices.

The dependent variables in the demand equations were the real retail prices of beef (PB/CPI) and pork (PP/CPI). Studies of US beef and pork demand agree that they are substitute products and their demands are affected by income (Chavas, 1983; Conway *et al.*, 1987; Freebairn and Rausser, 1975; Ladd and Karg, 1973). Most studies have found that beef and pork prices are affected by broiler price or supply or consumption. The explanatory variables in our demand equations were the per capita quantities of beef and pork (QB and QP), real per capita disposable income (RY), a dummy variable PR73, and the seasonal dummy variables. PR73 shifted the intercept for the months in 1973 that the beef price ceiling was in effect. When we included broiler consumption in the demand equations, we encountered serious multicollinearity problems; consequently, we excluded broiler consumption from equations reported here. One estimated pork demand equation is presented in Table 1. The estimation procedure corrected for autocorrelated errors and heteroscedasticity. An F-ratio indicated significant changes in the intercept and the coefficients of QB, QP, and RY. The coefficients changed by 68, 59, and 96%.

We based our marketing margin equations on previous empirical work (Freebairn and Rausser, 1975; Heien, 1980; Ikerd, 1984; Ladd and Karg, 1973; Lamm and Westcott, 1981) and on firm theory. Following previous empirical studies, we incorporated a mark-up-pricing hypothesis, i.e., selling price equals buying price plus a mark-up. The mark-up is affected by prices of inputs. Recently Noteboom (1986) applied a mark-up pricing hypothesis

Table 1. *Estimated pork demand and farm–wholesale pork margin equations*

Variables	Pork demand		Farm–wholesale pork margin (FWMP)	
	Period 1	Period 2	Period 1	Period 2
Intercept	44.050**	11.291	4.594**	19.871**
QB	1.027**	0.329		
QP	−2.103**	−0.863**		
RY	8.112**	15.946*		
PR73	4.284**			
FWMB			0.312	0.853**
FVP[a]			0.115**	0
CI			0.096**	0.019
Seasonal dummies	yes		yes	

[a]Second period coefficient set equal to zero. When FVP was included in equation for period two, it had a small t value.
*$\alpha < 0.10$.
**$\alpha < 0.05$.

and included operating costs in a study of margins of individual retail shops. Previous margin studies have derived their equations from theory of a single-product firm and have treated each margin as being determined independently of other margins: the exception is Ladd and Karg (1973). Holdren (1960), however, has shown that margins are interrelated in multiproduct firms. Because red meats are handled by multiproduct firms we allowed for interdependence between beef and pork margins.

A static farm–wholesale pork marketing margin (FWMP) equation is presented in Table 1. FWMB is the farm–wholesale margin on beef, FVP is the farm value of pork, and CI is an index of prices of inputs used in processing and marketing meat. The two stage least squares estimation of this equation was corrected for the presence of autocorrelated errors and heteroscedasticity. An F-ratio indicated significant coefficient changes for all except the seasonal dummies. The coefficients of FWMB and CI changed by 173 and 80%. We also estimated a wholesale–retail pork margin (WRMP) equation. Results indicated no structural change in that equation between periods 1 and 2. That equation was

$$\underset{}{\mathrm{WRMP}_t} = \underset{(-4.45)}{-10.311} + \underset{(3.23)}{0.286}\ \mathrm{WRMB} + \underset{(3.86)}{0.112}\ \mathrm{CI3}$$

where WRMP is the wholesale–retail margin on beef and CI3 is an index of input prices.

Some other equations that we estimated were superior on economic grounds to the ones presented here. Some were more satisfactory on statistical criteria also, e.g., adjusted R^2, t-ratios. We chose these for presentation because (a) they are simpler to present and discuss, and (b) they tell the same stories about effects of structural change upon margins and about interrelations between beef and pork margins as do the more complex equations.

The reduced form of the margin equations (line three of Equation 8) is (ignoring the residual)

$$\mathbf{Y}_{3s} = E_{3s}^{-1}(F_{3s}\mathbf{X}_s - E_{2s}\mathbf{Y}_{2s}) \tag{9}$$

This equation was decomposed into the effects of changes in coefficients, variables' means,

and coefficient/variable mean interactions, as in Equation 1. The decompositions for the farm–wholesale pork margin are presented in Table 2. Line one of Table 2 shows the effect of changes in margin coefficients on the FWMP and mathematically is $(E_{32}^{-1} F_{32} - E_{31}^{-1} F_{31}) \bar{X}_1 - (E_{32}^{-1} E_{22} - E_{31}^{-1} E_{21}) \bar{Y}_{21}$. The second line is the effect of changes in the means of both retail prices and the exogenous variables of the margin equations. Mathematically, this quantity is $E_{31}^{-1} F_{31}(\bar{X}_2 - \bar{X}_1) - E_{31}^{-1} E_{21} (\bar{Y}_{22} - \bar{Y}_{21})$.

Table 2. *Effect of changes in margin coefficients, variables' means, and interactions upon the farm–wholesale pork margin*

Source	Effect on FWMP[a] (cents per pound)
Margin coefficients	2.12
Margin variables' means	14.86
Total interaction[b]	−10.60

[a]Total change in FWMP = 6.38 cents per pound.
[b]Includes effect of non-zero residual.

Despite significant and substantial changes in the coefficients of the farm–wholesale pork margin equation, these changes had a relatively small effect on the change in the level of this margin. The effect of the changes in the means of the exogenous variables was quite important.

The mean decomposition described in Equation 1 was also applied to the change in nominal retail pork price (PP). Substituting the reduced form in real prices (line one of Equation 8) into the identity in the second line of Equation 8, we found the reduced form in nominal retail prices (ignoring residuals)

$$\mathbf{Y}_{2s} = D_{2s}^{-1} F_{1s} X_s \tag{10}$$

The first column of Table 3 presents the results of this decomposition. The total change in the mean retail pork price was 56.58 cents per pound. Increases in the means of the intercept (CPI) and income had large effects on the change in the pork price.

The decomposition technique also allowed us to isolate the effect of changes in the demand equations on the margins. We calculated the effect on the farm–wholesale pork margin from the changes in coefficients, variables' means, and coefficient/variable mean interactions in the pork demand equation. We substituted Equation 10 into Equation 9 and decomposed the result as in Equation 1, allowing only D_{2s}, F_{1s}, and the demand variables within X_s to change between the two periods. These effects are presented in column two of Table 3.

We also calculated the reliability of the estimated change in the mean of the pork price. Equation 3 was applied to Equation 10 to arrive at the variance of the change in the nominal mean retail pork price. The total change in the pork price between the two sample periods was 56.58 cents per pound, and the variance was calculated to be 23.83. A 95% confidence interval about the mean change was then (47.01, 66.15).

Table 3. *Effect of changes in coefficients, variables' means, and interaction in pork demand equation upon retail pork price and farm–wholesale pork margin*

Source	Effect on nominal PP[a]	Effect on FWMP[b] (cents per pound)
Coefficients		
Intercept	−44.91	−4.87
QB	−9.39	−1.02
QP	8.94	0.97
RY	33.56	3.64
Means[c]		
Intercept	53.07	5.76
QB	9.61	1.04
QP	−14.93	−1.62
RY	35.04	3.80
PR73	−0.33	−0.04
Seasonal dummies	−0.39	−0.04
Total interaction	−13.69	−1.48

[a]Total change in PP = 56.58 cents per pound.
[b]Change in FWMP due to the change in retail pork price alone is 6.14 cents per pound.
[c]Variables are multiplied by the Consumer Price Index (CPI).

Finally, we calculated the change in the variance between the two sample periods for the nominal pork price equation. Each of the four terms in $\mathrm{d}V(y)$ (Equation 6) was calculated directly.

$$\sum_i \sum_j C(x_{i1}, x_{j1}) d(b_i b_j) = -60.52$$

$$\sum_i \sum_j b_{i1} b_{j1} \mathrm{d}C(x_i, x_j) = 265.27$$

$$\sum_i \sum_j d(b_i b_j) \mathrm{d}C(x_i x_j) = -39.26$$

From the sum of the three terms, $\mathrm{d}V(\mathrm{PP}) = 165.49$. The change in the variance due to the change in the coefficients (in absolute value) was small relative to the change in the variance due to the change in the covariances among the x's.

The change in the residual variance, $\mathrm{d}V(e)$, equalled 180.61 and the change in the variance of pork price, $\mathrm{d}V(\mathrm{PP})$, equalled 346.10. More than half the change in the variance of retail pork price was unexplained: it was due to the change in the residual variance.

IV. CONCLUSION

Identifying structural change in coefficients in econometric studies is only the first step in understanding how the changes have affected the level of the endogenous variable of interest.

After all, initial levels and changes in coefficients as well as initial levels and changes in the exogenous variables determine the change in the endogenous variable. One could be in error if, based on significant and 'large' coefficient changes, one were to conclude that these coefficient changes were largely responsible for the changes in the endogenous variable. This paper outlined procedures one can use to identify the 'economic' significance of coefficient structural change.

In the numerical example, statistically significant coefficient changes were identified in both the pork demand equation and the farm–wholesale pork margin equation. Coefficients changed by amounts ranging from 59 to 173%. Large as they were, these coefficient changes had relatively little to do with the changes that took place in the levels of the pork price and the farm–wholesale margin, and in the variances of the pork price and pork margin. The numerical example illustrates that the economic significance of statistically significant changes in coefficients can be quite small.

We also identify a specification error in margin studies that treat each margin as being determined independently of margins on other products. We found beef and pork margins to be interdependent.

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REFERENCES

Brown, R. L., Durbin, J. and Evans, J. M. (1975) Techniques for testing the consistency of regression relationships over time, *Journal of the Royal Statistical Society* (Series B), **37**, 149–92.

Chavas, J. P. (1983) Structural change in the demand for meat, *American Journal of Agricultural Economics*, **65**, 148–53.

Chow, G. C. (1965) A test of equality between sets of observations in two linear regressions, *Econometrica*, **28**, 591–605.

Conway, R. K., Hallahan, G. B., Stillman, R. P. and Prentice, P. T. (1987) Forecasting livestock prices: fixed and stochastic coefficients estimation, US Department of Agriculture, Economic Research Service, Technical Bulletin 1725, Washington, DC.

Dhrymes, P. J., Howrey, E. P., Hymans, S. H., Kmenta, J., Leamer, E. E., Quandt, R. E., Ramsey, J. B., Shapiro, H. T. and Zarnowitz, V. (1972) Criteria for evaluation of econometric models, *Annals of Economic and Social Measurement*, **1**, 291–324.

Fisher, B. S. (1982) Rational expectations in agricultural economics research and policy analysis, *American Journal of Agricultural Economics*, **64**, 260–5.

Freebairn, J. W. and Rausser, G. C. (1975) Effects of changes in the level of U.S. beef imports, *American Journal of Agricultural Economics*, **57**, 676–88.

Girshick, M. A. and Haavelmo, T. (1947) Statistical analysis of the demand for food: examples of simultaneous estimation of structural equations, *Econometrica*, **15**, 79–110.

Heien, D. (1980) Markup pricing in a dynamic model of the food industry, *American Journal of Agricultural Economics*, **62**, 10–18.

Holdren, B. R. (1960) *The Structure of a Retail Market and the Market Behavior of Retail Units.* Prentice Hall, Englewood Cliffs, NJ.

Ikerd, J. E. (1984) Forecasting spreads between retail beef and live cattle prices: a model for routine use by market analysts, in *Proceedings of NCR-134 Conference on Applied Commodity Price Analysis and Forecasting*, 26–27 April 1984, pp. 301–18.

Koopmans, T. C. (1949) Identification in economic model construction, *Econometrica*, **17**, 125–44.

Ladd, G. W. and Karg, G. (1973) Econometric and programming analyses of the beef–pork marketing sector, Iowa State University, Agriculture and Home Economics Experiment Station Research Bulletin 577, Ames, Iowa.

Lamm, R. M. and Westcott, P. C. (1981) The effects of changing input costs on food prices, *American Journal of Agricultural Economics*, **63**, 187–96.

Lucas, R. E. Jr (1976) Economic policy evaluation: a critique, *Journal of Monetary Economics Supplement*, **3**, 19–46.

Marschak, J. (1947) Economic structure, path, prediction, and policy: abstract of paper presented at Jan. 24–27, 1947, meetings of Econometric Society, *Econometrica*, **15**, 153–4.

Mellor, C. J. and Hessner, C. (1986) Structural change in the yellow fats market, *Applied Economics*, **18**, 663–73.

Noteboom, B. (1985) A mark-up model of retail margins, *Applied Economics*, **17**, 647–67.

Rausser, G. C., Mundlak, Y. and Johnson, S. R. (1982) Structural change, updating, and forecasting, in *New Directions in Econometric Modeling and Forecasting in U.S. Agriculture* (Ed.) G. C. Rausser, North-Holland, New York, Amsterdam, pp. 659–718.

Sheflin, N. (1984) Transition function estimation of structural shifts in models of American trade union growth, *Applied Economics*, **16**, 73–80.